1998

CALCULUS BC

SECTION I, Part A

Time — 55 minutes

Number of questions — 28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

2. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

3. The slope of the line tangent to the curve $y^2 + (xy + 1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

4. $\int \frac{1}{x^2 - 6x + 8}\,dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

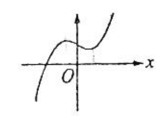
(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

(E) $\ln\left|(x-2)(x-4)\right| + C$

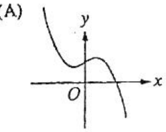
5. If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

(A) $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$
(B) $f''(g(x))g'(x) + f'(g(x))g''(x)$
(C) $f''(g(x))[g'(x)]^2$
(D) $f''(g(x))g''(x)$
(E) $f''(g(x))$



6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

(A)



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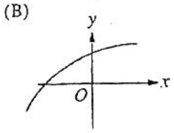

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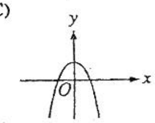

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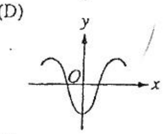
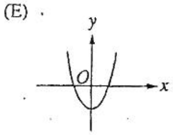


(E)

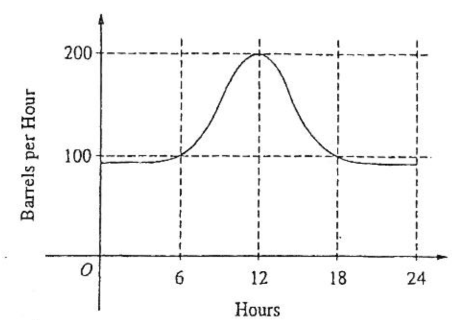


7. $\int_1^e \left(\frac{x^2 - 1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

8. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

10. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^3 - t$ and its y-coordinate is $(2t - 1)^3$. The acceleration vector of the particle at $t = 1$ is

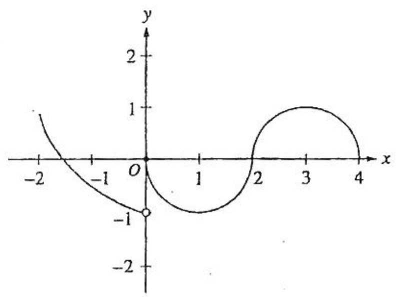
(A) $(0, 1)$ (B) $(2, 3)$ (C) $(2, 6)$ (D) $(6, 12)$ (E) $(6, 24)$

11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

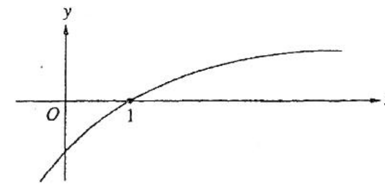
(A) $1 - \frac{1}{2} + \frac{1}{24}$
(B) $1 - \frac{1}{2} + \frac{1}{4}$
(C) $1 - \frac{1}{3} + \frac{1}{5}$
(D) $1 - \frac{1}{4} + \frac{1}{8}$
(E) $1 - \frac{1}{6} + \frac{1}{120}$

15. $\int x \cos x\,dx =$

(A) $x \sin x - \cos x + C$
(B) $x \sin x + \cos x + C$
(C) $-x \sin x + \cos x + C$
(D) $x \sin x + C$
(E) $\frac{1}{2}x^2 \sin x + C$

16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

19. The area of the region inside the polar curve $r = 4 \sin \theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4 \sin \theta - 2)^2\,d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4 \sin \theta - 2)^2\,d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 \sin \theta - 2)^2\,d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{5\pi}{6}} (16 \sin^2\theta - 4)\,d\theta$ (E) $\frac{1}{2}\int_0^{\pi} (16 \sin^2\theta - 4)\,d\theta$

20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1}\,dt$
(B) $\int_0^1 \sqrt{t^2 + t}\,dt$
(C) $\int_0^1 \sqrt{t^4 + t^2}\,dt$
(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4}\,dt$
(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\,dt$

22 $\lim_{x \to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

23 If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

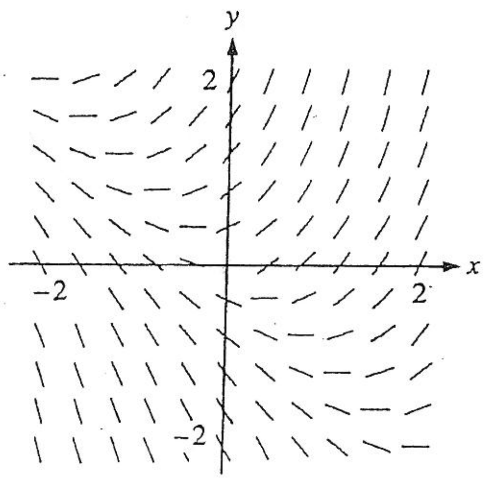
(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

24 Let f be a function defined and continuous on the closed interval $[a, b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements must be true?

I. $f'(c)$ exists.
II. If $f'(c)$ exists, then $f'(c) = 0$.
III. If $f''(c)$ exists, then $f''(c) \le 0$.

(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only



25. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

26 ~~25~~. $\int_0^{\infty} x^2 e^{-x^3}\, dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

27 The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

28 If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n^{\,n-1}$

CALCULUS BC

SECTION I, Part B

Time — 50 minutes

Number of questions — 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

29\. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

30\. If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$

(D) $(e^{-t}, \cos t)$ (E) $(e^{-t}, -\cos t)$

31\. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$ (C) $-\frac{(0.1)C}{2\pi}$ (D) $(0.1)^2C$ (E) $(0.1)^2\pi C$

(B) $-(0.1)C$

32\. Let f be the function given by $f(x) = \frac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

(A) None
(B) 1 only
(C) 2 only
(D) 4 only
(E) 1 and 4 only

33\. Let R be the region enclosed by the graph of $y = 1 + \ln(\cos^4 x)$, the x-axis, and the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

34\. If $\frac{dy}{dx} = \sqrt{1-y^2}$, then $\frac{d^2y}{dx^2} =$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

35\. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx =$

(A) $2\int_3^5 g(x)\,dx + 7$ (C) $2\int_3^5 g(x)\,dx + 28$ (E) $\int_3^5 g(x)\,dx + 14$

(B) $2\int_3^5 g(x)\,dx + 14$ (D) $\int_3^5 g(x)\,dx + 7$

36\. The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

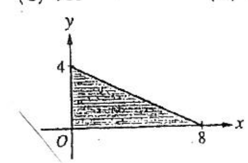
37\. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

x	2	5	7	8
$f(x)$	10	30	40	20

38\. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

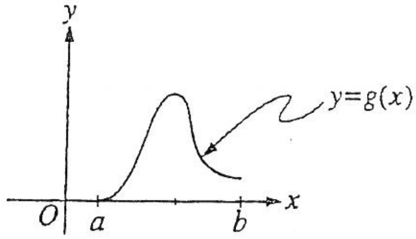


39\. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

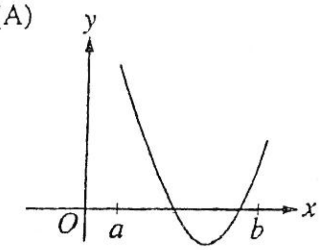
(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

40\. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$



41\. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


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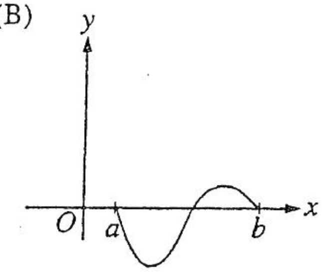

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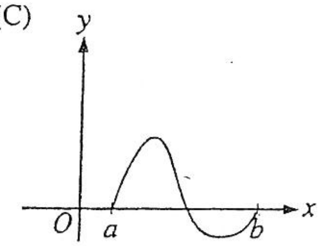

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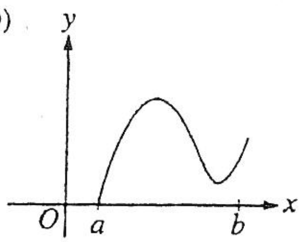

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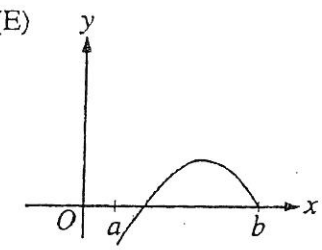
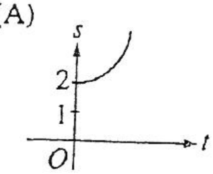


42\. The graph of the function represented by the Maclaurin series

$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

43\. A particle starts from rest at the point (2, 0) and moves along the x-axis with a constant positive acceleration for time $t \ge 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t ?

(A)


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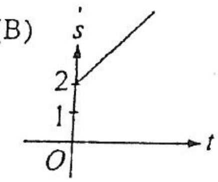
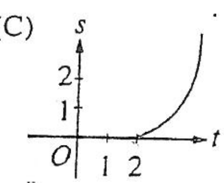
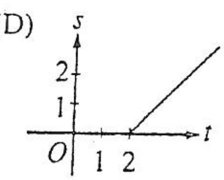


(C)


(D)


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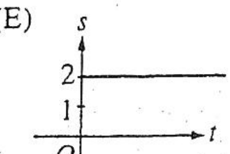

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

44\. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

45\. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5 ?

(A) 2.4 (B) 2.5