2nd Deriv test Ans (SWOK)

[4] $f'(x) = 16x - 8x^3 = 8x(2 - x^2) = 0 \Leftrightarrow x = 0, \pm\sqrt{2}$. $f''(x) = 16 - 24x^2$.
$f''(0) = 16 > 0 \Rightarrow f(0) = 0$ is a *LMIN*. $f''(\pm\sqrt{2}) = -32 < 0 \Rightarrow$
$f(\pm\sqrt{2}) = 8$ are *LMAX*. $f''(x) > 0$ and f is *CU* on $(-\sqrt{2/3}, \sqrt{2/3})$.
$f''(x) < 0$ and f is *CD* on $(-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty)$. *PI* at $x = \pm\sqrt{2/3}$.

[5] $f'(x) = 12x^5 - 24x^3 = 12x^3(x^2 - 2) = 0 \Leftrightarrow x = 0, \pm\sqrt{2}$.
$f''(x) = 60x^4 - 72x^2 = 60x^2(x^2 - \frac{6}{5})$. $f''(\pm\sqrt{2}) = 96 > 0 \Rightarrow f(\pm\sqrt{2}) = -8$ are *LMIN*. $f''(0) = 0$ gives no information. By the first derivative test, $f(0) = 0$ is a *LMAX*. Since $60x^2 \geq 0$, the sign of f'' is determined by $(x^2 - \frac{6}{5})$, it follows that $f''(x) > 0$ if $|x| > \sqrt{6/5}$ and $f''(x) < 0$ if $|x| < \sqrt{6/5}$ $(x \neq 0)$. Thus, f is *CU* on $(-\infty, -\sqrt{6/5}) \cup (\sqrt{6/5}, \infty)$ and *CD* on $(-\sqrt{6/5}, \sqrt{6/5})$. *PI* at $x = \pm\sqrt{6/5}$.

[7] $f'(x) = 4x(x^2 - 1) = 0 \Leftrightarrow x = 0, \pm 1$. $f''(x) = 4(3x^2 - 1)$.
$f''(0) = -4 < 0 \Rightarrow f(0) = 1$ is a *LMAX*. $f''(\pm 1) = 8 > 0 \Rightarrow f(\pm 1) = 0$ are *LMIN*.
$f''(x) > 0$ and f is *CU* on $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$.
$f''(x) < 0$ and f is *CD* on $(-\sqrt{1/3}, \sqrt{1/3})$. *PI* at $x = \pm\sqrt{1/3}$

[8] $f'(x) = 4x^2(x - 3) = 0 \Leftrightarrow x = 0, 3$. $f''(x) = 12x(x - 2)$.
$f''(3) = 36 > 0 \Rightarrow f(3) = -17$ is a *LMIN*. $f''(0) = 0$ gives no information.
By the first derivative test, $f(0) = 10$ is not a local extremum.
The sign of f'' changes at $x = 0, 2$. $f''(x) > 0$ and f is *CU* on $(-\infty, 0) \cup (2, \infty)$.
$f''(x) < 0$ and f is *CD* on $(0, 2)$. *PI* at $x = 0, 2$.

[10] $f'(x) = -\frac{2}{3}x^{-1/3}$ is undefined when $x = 0$. $f''(x) = \frac{2}{9}x^{-4/3}$. Since $f''(0)$ is undefined, use the first derivative test to show that $f(0) = 2$ is a *LMAX*.
Since $f''(x) > 0$, f is *CU* on $(-\infty, 0)$ and $(0, \infty)$. There are no *PI*.

[11] $f'(x) = \dfrac{5(3x + 4)}{3x^{1/3}} = 0 \Leftrightarrow x = -\frac{4}{3}$. f' fails to exist at $x = 0$. $f''(x) = \dfrac{10(3x - 2)}{9x^{4/3}}$.
$f''(-\frac{4}{3}) < 0$ and $f(-\frac{4}{3}) = 4\sqrt[3]{6} \approx 7.27$ is a *LMAX*. Since $f''(0)$ is undefined, use the first derivative test to show that $f(0) = 0$ is a *LMIN*. Since $9x^{4/3} \geq 0$, the sign of f'' is determined by $(3x - 2)$. Thus, $f''(x) < 0$ and f is *CD* on $(-\infty, 0) \cup (0, \frac{2}{3})$ and
$f''(x) > 0$ and f is *CU* on $(\frac{2}{3}, \infty)$. *PI* at $x = \frac{2}{3}$.
Note: f is not *CD* at $x = 0$ since no tangent line exists at $x = 0$.

[14] $f'(x) = \dfrac{2(2x + 1)}{(3x + 2)^{2/3}} = 0 \Leftrightarrow x = -\frac{1}{2}$. f' fails to exist at $x = -\frac{2}{3}$.
$f''(x) = \dfrac{4(x + 1)}{(3x + 2)^{5/3}}$. $f''(-\frac{1}{2}) = 4\sqrt[3]{4} > 0 \Rightarrow f(-\frac{1}{2}) = -\frac{1}{4}\sqrt[3]{4} \approx -0.4$ is a *LMIN*.
By the first derivative test, $f(-\frac{2}{3}) = 0$ is not an extremum.
$f''(x) > 0$ and f is *CU* on $(-\infty, -1)$ and $(-\frac{2}{3}, \infty)$.
$f''(x) < 0$ and f is *CD* on $(-1, -\frac{2}{3})$. x-coordinates of *PI* are -1 and $-\frac{2}{3}$.

[16] $f'(x) = \dfrac{3x + 6}{2\sqrt{x}} = 0 \Leftrightarrow x = -2$, which is not in the domain of f. f' fails to exist at $x = 0$. $f(0) = 0$ is an endpoint extremum since $f > 0$ for $x > 0$. There are no local extrema. $f''(x) = \dfrac{3x - 6}{4x^{3/2}}$. $f''(x) < 0$ for $0 < x < 2$ and f is *CD* on $(0, 2)$.
$f''(x) > 0$ for $x > 2$ and f is *CU* on $(2, \infty)$. x-coordinate of *PI* is 2. See *Figure 16*.

[17] $f'(x) = \dfrac{3x(6 - x^2)}{\sqrt{9 - x^2}} = 0 \Leftrightarrow x = 0, \pm\sqrt{6}$. f' fails to exist at $x = \pm 3$, which are endpoints of the domain. $f''(x) = \dfrac{3(54 - 27x^2 + 2x^4)}{(9 - x^2)^{3/2}}$. $f''(\pm\sqrt{6}) = -12\sqrt{3} < 0 \Rightarrow$
$f(\pm\sqrt{6}) = 6\sqrt{3} \approx 10.4$ are *LMAX*. $f''(0) = 6 > 0 \Rightarrow f(0) = 0$ is a *LMIN*.
$f''(x) = 0 \Rightarrow x = \pm\frac{1}{2}\sqrt{27 \pm 3\sqrt{33}} \Rightarrow x = \pm\frac{1}{2}\sqrt{27 - 3\sqrt{33}}$ for $|x| < 3$.
Let $a = -\frac{1}{2}\sqrt{27 - 3\sqrt{33}} \approx -1.56$ and $b = -a$. $f''(x) > 0$ and f is *CU* on (a, b).
$f''(x) < 0$ and f is *CD* on $(-3, a)$ and $(b, 3)$. x-coordinates of *PI* are a and b.

[19] The *CN* are $x = \frac{\pi}{4}, \frac{5\pi}{4}$. $f''(x) = -\cos x - \sin x$. $f''(\frac{\pi}{4}) = -\sqrt{2} < 0 \Rightarrow$
$f(\frac{\pi}{4}) = \sqrt{2}$ is a *LMAX*. $f''(\frac{5\pi}{4}) = \sqrt{2} > 0 \Rightarrow f(\frac{5\pi}{4}) = -\sqrt{2}$ is a *LMIN*.

[20] The *CN* are $x = \frac{3\pi}{4}, \frac{7\pi}{4}$. $f''(x) = -\cos x + \sin x$. $f''(\frac{7\pi}{4}) = -\sqrt{2} < 0 \Rightarrow$
$f(\frac{7\pi}{4}) = \sqrt{2}$ is a *LMAX*. $f''(\frac{3\pi}{4}) = \sqrt{2} > 0 \Rightarrow f(\frac{3\pi}{4}) = -\sqrt{2}$ is a *LMIN*.

[25] The only *CN* in $(-\frac{\pi}{2}, \frac{\pi}{2})$ is $x = 0$.
$f''(x) = \frac{1}{2}(\frac{1}{2}\sec\frac{1}{2}x\tan^2\frac{1}{2}x + \frac{1}{2}\sec^3\frac{1}{2}x) \Rightarrow f''(0) = \frac{1}{4} > 0 \Rightarrow f(0) = 1$ is a *LMIN*.

[26] The only *CN* in $(\frac{\pi}{6}, \frac{5\pi}{6})$ is $x = \frac{3\pi}{4}$. $f''(x) = 4\csc^2 x\cot x(\cot x + 1) + 2\csc^4 x \Rightarrow$
$f''(\frac{3\pi}{4}) = 8 > 0 \Rightarrow f(\frac{3\pi}{4}) = -1$ is a *LMIN*.

[27] The only *CN* in $(-\frac{\pi}{3}, \frac{\pi}{3})$ is $x = \frac{\pi}{4}$. $f''(x) = 4\sec^2 x\tan x(1 - \tan x) - 2\sec^4 x \Rightarrow$
$f''(\frac{\pi}{4}) = -8 < 0 \Rightarrow f(\frac{\pi}{4}) = 1$ is a *LMAX*.

[28] The only *CN* in $(-\frac{\pi}{4}, \frac{\pi}{4})$ is $x = \frac{\pi}{6}$.
$f''(x) = \sec x\tan x(\sec x - 2\tan x) + \sec x(\sec x\tan x - 2\sec^2 x) \Rightarrow$
$f''(\frac{\pi}{6}) = -\frac{4}{\sqrt{3}} < 0$, $f(\frac{\pi}{6}) = -\sqrt{3}$ is a *LMAX*.