4.1 - Antiderivatives

$f'(x) = x^2$

$f(x) = \frac{1}{3}x^3 + 7$ $\quad \frac{1}{3}x^3 - 5\sqrt[3]{571}$

$\frac{1}{3}x^3 + \pi$

$f(x) = \frac{1}{3}x^3 + C$ ← arbitrary constant

$f(x)$ is the antiderivative of $f'(x)$.

Definition: A function F is an antiderivative of f on an interval if $F'(x) = f(x)$ for every x on the interval.

$$\int f(x)\,dx = F(x) + C$$

- $\int$: integral sign ("indefinite integral")
- $f(x)$: integrand
- dx: variable of integration
- $F(x)$: antideriv.
- C: arbitrary constant

Rules:

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C = \frac{x^{n+1}}{n+1} + C$ $\quad (n \neq -1)$

$\int x^5 dx = \frac{1}{6}x^6 + C$ $\quad \int x^{-4} dx = -\frac{1}{3}x^{-3} + C$

$\int \sqrt{x}\,dx = \int x^{1/2} dx = \frac{2}{3}x^{3/2} + C$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int 5x^3dx = 5\int x^3dx = 5\cdot\frac{1}{4}x^4+c = \boxed{\frac{5}{4}x^4+c}$$

$$\int f(x)\pm g(x)\,dx = \int f(x)dx \pm \int g(x)dx$$

$$\int\left(x^4-2x^2+3\right)dx = \boxed{\frac{1}{5}x^5-\frac{2}{3}x^3+3x+c}$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \csc^2 x\,dx = -\cot x + c$$

$$\int \sec x\tan x\,dx = \sec x + c$$

$$\int \csc x\cot x\,dx = -\csc x + c$$

Ex. $\int \frac{(x^2-1)^2}{x^2}dx = \int \frac{x^4-2x^2+1}{x^2}dx = \int x^2-2+x^{-2}dx$

$-2x^0$

$$= \boxed{\frac{1}{3}x^3-2x^1-\frac{1}{x}+c}$$

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Solving Differential Equations

* Given a deriv eqn. (or 2nd deriv) and finding orig. function.

Given $\frac{dy}{dx} = 6x^2 + x - 5$ and $(0, 2)$ (x, y) on solutions curve.

a) Find general solution b) Find particular solution.

$$\int 6x^2 + x - 5 \, dx = 2x^3 + \frac{1}{2}x^2 - 5x + C \Rightarrow \text{general solution}$$

$$y = 2x^3 + \frac{1}{2}x^2 - 5x + C$$

$$2 = 2(0)^3 + \frac{1}{2}(0)^2 - 5(0) + C$$

$$C = 2$$

$$y = 2x^3 + \frac{1}{2}x^2 - 5x + 2 \rightarrow \text{particular solution}$$

Ex 2. $y'' = \sin x + \cos x$ $\quad y(0) = 3$ $\quad y'(0) = -2$

$$y' = \int \sin x + \cos x \, dx = -\cos x + \sin x + C$$

$$-2 = -\cos(0) + \sin(0) + C$$

$$-2 = -1 + 0 + C \qquad C = -1$$

$$y' = -\cos x + \sin x - 1$$

$$y = \int (-\cos x + \sin x - 1) \, dx = -\sin x - \cos x - x + C_1$$

$$3 = -\sin 0 - \cos 0 - 0 + C_1, \qquad 3 = -1 + C_1$$

$$C_1 = 4$$

$$y = -\sin x - \cos x - x + 4$$

Projectile Motion

A ball is thrown (up) off a 30 ft tall building with initial speed of 40 ft/s. Find height of ball @ any time t.

$v(0)$ = positive

$s(0) = 30$ ft.

$v(0) = s'(0) = 40$ ft/s

Projectile ⇒ acceleration is constant ⇒ gravity only!!

~~A~~ $a(t) = -32 \text{ ft/s}^2 = -9.8 \text{ m/s}^2$

$$v(t) = \int -32\,dt = -32t + c$$

$$v(0) = 40 = -32(0) + c \qquad c = 40$$

$$v(t) = -32t + 40$$

$$s(t) = \int -32t + 40\,dt = -16t^2 + 40t + c_1$$

$$s(0) = 30 = -16(0)^2 + 40(0) + c_1 \qquad c_1 = 30$$

$$\boxed{s(t) = -16t^2 + 40t + 30}$$