Exer. 1–42: Evaluate. 4.1 Practice

#1-41 odd, 49, 54, 55, 59, 6

1 $\int (4x + 3)\, dx$

2 $\int (4x^2 - 8x + 1)\, dx$

3 $\int (9t^2 - 4t + 3)\, dt$

4 $\int (2t^3 - t^2 + 3t - 7)\, dt$

$\int \left(\frac{1}{z^3} - \frac{3}{z^2}\right) dz$

6 $\int \left(\frac{4}{z^7} - \frac{7}{z^4} + z\right) dz$

7 $\int \left(3\sqrt{u} + \frac{1}{\sqrt{u}}\right) du$

8 $\int (\sqrt{u^3} - \frac{1}{2}u^{-2} + 5)\, du$

9 $\int (2v^{5/4} + 6v^{1/4} + 3v^{-4})\, dv$

10 $\int (3v^5 - v^{5/3})\, dv$

11 $\int (3x - 1)^2\, dx$

12 $\int \left(x - \frac{1}{x}\right)^2 dx$

13 $\int x(2x + 3)\, dx$

14 $\int (2x - 5)(3x + 1)\, dx$

15 $\int \frac{8x - 5}{\sqrt[3]{x}}\, dx$

16 $\int \frac{2x^2 - x + 3}{\sqrt{x}}\, dx$

17 $\int \frac{x^3 - 1}{x - 1}\, dx, \quad x \neq 1$

18 $\int \frac{x^3 + 3x^2 - 9x - 2}{x - 2}\, dx, \quad x \neq 2$

19 $\int \frac{(t^2 + 3)^2}{t^6}\, dt$

20 $\int \frac{(\sqrt{t} + 2)^2}{t^3}\, dt$

21 $\int \frac{3}{4} \cos u\, du$

22 $\int -\frac{1}{5} \sin u\, du$

23 $\int \frac{7}{\csc x}\, dx$

24 $\int \frac{1}{4 \sec x}\, dx$

25 $\int (\sqrt{t} + \cos t)\, dt$

26 $\int (\sqrt[3]{t^2} - \sin t)\, dt$

27 $\int \frac{\sec t}{\cos t}\, dt$

28 $\int \frac{1}{\sin^2 t}\, dt$

29 $\int (\csc v \cot v \sec v)\, dv$

30 $\int (4 + 4 \tan^2 v)\, dv$

31 $\int \frac{\sec w \sin w}{\cos w}\, dw$

32 $\int \frac{\csc w \cos w}{\sin w}\, dw$

33 $\int \frac{(1 + \cot^2 z) \cot z}{\csc z}\, dz$

34 $\int \frac{\tan z}{\cos z}\, dz$

35 $\int D_x \sqrt{x^2 + 4}\, dx$

36 $\int D_x \sqrt[3]{x^3 - 8}\, dx$

37 $\int \frac{d}{dx} (\sin \sqrt[3]{x})\, dx$

38 $\int \frac{d}{dx} (\sqrt{\tan x})\, dx$

39 $D_x \int (x^3 \sqrt{x - 4})\, dx$

40 $D_x \int (x^4 \sqrt[3]{x^2 + 9})\, dx$

41 $\frac{d}{dx} \int \cot x^3\, dx$

42 $\frac{d}{dx} \int \cos \sqrt{x^2 + 1}\, dx$

Exer. 43–48: Evaluate the integral if a and b are constants.

43 $\int a^2\, dx$

44 $\int ab\, dx$

45 $\int (at + b)\, dt$

46 $\int \left(\frac{a}{b^2}\, t\right) dt$

47 $\int (a + b)\, du$

48 $\int (b - a^2)\, du$

Exer. 49–56: Solve the differential equation subject to the given conditions.

49 $f'(x) = 12x^2 - 6x + 1; \quad f(1) = 5$

50 $f'(x) = 9x^2 + x - 8; \quad f(-1) = 1$

51 $\frac{dy}{dx} = 4x^{1/2}; \quad y = 21$ if $x = 4$

52 $\frac{dy}{dx} = 5x^{-1/3}; \quad y = 70$ if $x = 27$

53 $f''(x) = 4x - 1; \quad f'(2) = -2; \quad f(1) = 3$

54 $f''(x) = 6x - 4; \quad f'(2) = 5; \quad f(2) = 4$

55 $\frac{d^2y}{dx^2} = 3 \sin x - 4 \cos x; \quad y = 7$ and $y' = 2$ if $x = 0$

56 $\frac{d^2y}{dx^2} = 2 \cos x - 5 \sin x; \quad y = 2 + 6\pi$ and $y' = 3$ if $x = \pi$

Exer. 57–58: If a point is moving on a coordinate line with the given acceleration $a(t)$ and initial conditions, find $s(t)$.

57 $a(t) = 2 - 6t; \quad v(0) = -5; \quad s(0) = 4$

58 $a(t) = 3t^2; \quad v(0) = 20; \quad s(0) = 5$

59 A projectile is fired vertically upward from ground level with a velocity of 1600 ft/sec. Disregarding air resistance, find

(a) its distance $s(t)$ above ground at time t

(b) its maximum height

60 An object is dropped from a height of 1000 feet. Disregarding air resistance, find

(a) the distance it falls in t seconds

(b) its velocity at the end of 3 seconds

(c) when it strikes the ground

61 A stone is thrown directly downward from a height of 96 feet with an initial velocity of 16 ft/sec. Find

(b) If, after returning to Earth, the astronaut throws the same stone directly upward with the same initial velocity, find the maximum altitude.

63 If a projectile is fired vertically upward from a height of s_0 feet above the ground with a velocity of v_0 ft/sec, prove that if air resistance is disregarded, its distance $s(t)$ above the ground after t seconds is given by $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$, where g is a gravitational constant.

64 A ball rolls down an inclined plane with an acceleration of 2 ft/sec^2.

(a) If the ball is given no initial velocity, how far will it roll in t seconds?

(b) What initial velocity must be given for the ball to roll 100 feet in 5 seconds?

65 If an automobile starts from rest, what constant acceleration will enable it to travel 500 feet in 10 seconds?

66 If a car is traveling at a speed of 60 mi/hr, what constant (negative) acceleration will enable it to stop in 9 seconds?

67 A small country has natural gas reserves of 100 billion ft^3. If $A(t)$ denotes the total amount of natural gas consumed after t years, then dA/dt is the *rate of consumption*. If the rate of consumption is predicted to be $5 + 0.01t$ billion ft^3/year, in approximately how many years will the country's natural gas reserves be depleted?

68 Refer to Exercise 67. Based on U.S. Department of Energy statistics, the rate of consumption of gasoline in the United States (in billions of gallons per year) is approximated by $dA/dt = 2.74 - 0.11t - 0.01t^2$, with $t = 0$ corresponding to the year 1980. Estimate the number of gallons of gasoline consumed in the United States be-

[1] $\int (4x + 3)\,dx = 4\cdot\frac{x^2}{2} + 3x + C = 2x^2 + 3x + C$

[2] $\int (4x^2 - 8x + 1)\,dx = \frac{4}{3}x^3 - 4x^2 + x + C$

[3] $\int (9t^2 - 4t + 3)\,dt = 3t^3 - 2t^2 + 3t + C$

[4] $\int (2t^3 - t^2 + 3t - 7)\,dt = \frac{1}{2}t^4 - \frac{1}{3}t^3 + \frac{3}{2}t^2 - 7t + C$

[5] $\int \left(\frac{1}{z^3} - \frac{3}{z^2}\right) dz = -\frac{1}{2z^2} + \frac{3}{z} + C$

[6] $\int \left(\frac{4}{z^7} - \frac{7}{z^4} + z\right) dz = -\frac{2}{3z^6} + \frac{7}{3z^3} + \frac{1}{2}z^2 + C$

[7] $\int \left(3\sqrt{u} + \frac{1}{\sqrt{u}}\right) du = 2u^{3/2} + 2u^{1/2} + C$

[8] $\int (\sqrt{u^3} - \frac{1}{2}u^{-2} + 5)\,du = \frac{2}{5}u^{5/2} + \frac{1}{2}u^{-1} + 5u + C$

[9] $\int (2v^{5/4} + 6v^{1/4} + 3v^{-4})\,dv = \frac{8}{9}v^{9/4} + \frac{24}{5}v^{5/4} - v^{-3} + C$

[10] $\int (3v^5 - v^{5/3})\,dv = \frac{1}{2}v^6 - \frac{3}{8}v^{8/3} + C$

[11] $\int (3x - 1)^2\,dx = \int (9x^2 - 6x + 1)\,dx = 3x^3 - 3x^2 + x + C$

[12] $\int \left(x - \frac{1}{x}\right)^2 dx = \int (x^2 - 2 + x^{-2})\,dx = \frac{1}{3}x^3 - 2x - \frac{1}{x} + C$

[13] $\int x(2x + 3)\,dx = \int (2x^2 + 3x)\,dx = \frac{2}{3}x^3 + \frac{3}{2}x^2 + C$

[14] $\int (2x - 5)(3x + 1)\,dx = \int (6x^2 - 13x - 5)\,dx = 2x^3 - \frac{13}{2}x^2 - 5x + C$

[15] $\int \frac{8x - 5}{\sqrt[3]{x}}\,dx = \int (8x^{2/3} - 5x^{-1/3})\,dx = \frac{24}{5}x^{5/3} - \frac{15}{2}x^{2/3} + C$

[16] $\int \frac{2x^2 - x + 3}{\sqrt{x}}\,dx = \int (2x^{3/2} - x^{1/2} + 3x^{-1/2})\,dx = \frac{4}{5}x^{5/2} - \frac{2}{3}x^{3/2} + 6x^{1/2} + C$

[17] $\int \frac{x^3 - 1}{x - 1}\,dx = \int (x^2 + x + 1)\,dx \; (x \neq 1) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$

[18] $\int \frac{x^3 + 3x^2 - 9x - 2}{x - 2}\,dx = \int (x^2 + 5x + 1)\,dx \; (x \neq 2) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + x + C$

[19] $\int \frac{(t^2 + 3)^2}{t^6}\,dt = \int \frac{t^4 + 6t^2 + 9}{t^6}\,dt =$

$\int (t^{-2} + 6t^{-4} + 9t^{-6})\,dt = -t^{-1} - 2t^{-3} - \frac{9}{5}t^{-5} + C$

[20] $\int \frac{(\sqrt{t} + 2)^2}{t^3}\,dt = \int \frac{t + 4\sqrt{t} + 4}{t^3}\,dt =$

$\int (t^{-2} + 4t^{-5/2} + 4t^{-3})\,dt = -t^{-1} - \frac{8}{3}t^{-3/2} - 2t^{-2} + C$

[21] $\int \frac{3}{4}\cos u\,du = \frac{3}{4}\int \cos u\,du = \frac{3}{4}\sin u + C$

[22] $\int -\frac{1}{5}\sin u\,du = -\frac{1}{5}\int \sin u\,du = \frac{1}{5}\cos u + C$

[23] $\int \frac{7}{\csc x}\,dx = 7\int \sin x\,dx = -7\cos x + C$

[60] (a) $a(t) = -32 \Rightarrow v(t) = -32t + C$. $v(0) = 0 \Rightarrow C = 0$. $s(t) = -16t^2 + D$.

$s(0) = 1000 \Rightarrow D = 1000$ and $s(t) = -16t^2 + 1000$.

Hence, the object falls $16t^2$ feet in t seconds.

(b) $v(3) = -32(3) = -96$ ft/sec.

(c) When $s(t) = 0$, $-16t^2 + 1000 = 0$ and $t = \frac{5}{2}\sqrt{10} \approx 7.9$ sec.

[61] (a) $a(t) = -32 \Rightarrow v(t) = -32t + C$. $v(0) = -16 \Rightarrow C = -16$.

$s(t) = -16t^2 - 16t + D$. $s(0) = 96 \Rightarrow D = 96$.

(b) $s(t) = -16t^2 - 16t + 96 = 0$ when $t = 2$ (-3 is rejected).

(c) $v(2) = -32(2) - 16 = -80$ ft/sec.

[62] (a) For the general situation, $a(t) = -g \Rightarrow v(t) = -gt + C$. $v(0) = v_0 \Rightarrow$ $v(t) = -gt + v_0$. $s(t) = -\frac{1}{2}gt^2 + v_0t + D$. $s(0) = s_0 \Rightarrow s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$. Maximum altitude occurs when $v(t) = 0$. Since $g = 5.3$ and $v_0 = 60$, we have $v(t) = -5.3t + 60 = 0$ when $t = \frac{60}{5.3} \approx 11.32$.

$s(t) = -\frac{1}{2}(5.3)t^2 + 60t \Rightarrow s(\frac{60}{5.3}) \approx 339.6$ ft (since $s_0 = 0$).

(b) On earth, $g = 32 \Rightarrow t = \frac{60}{32} \Rightarrow s(\frac{60}{32}) = 56.25$ ft.

[63] See the solution of Exercise 62(a).

[64] (a) $a(t) = 2 \Rightarrow v(t) = 2t + C$. $v(0) = 0 \Rightarrow v(t) = 2t$. Thus, $s(t) = t^2 + D$.

In t seconds, the ball will roll a distance of $s(t) - s(0) = t^2$ ft.

If $v(0) = v_0$, then $s(t) = t^2 + v_0t + D$. The distance traveled in 5 sec is

$s(5) - s(0) = 5^2 + v_05 = 100$ if $v_0 = 15$ ft/sec.

[24] $\int \frac{1}{4\sec x}\,dx = \frac{1}{4}\int \cos x\,dx = \frac{1}{4}\sin x + C$

[25] $\int (\sqrt{t} + \cos t)\,dt = \frac{2}{3}t^{3/2} + \sin t + C$ [26] $\int (\sqrt[3]{t^2} - \sin t)\,dt = \frac{3}{5}t^{5/3} + \cos t + C$

[27] $\int \frac{\sec t}{\cos t}\,dt = \int \sec^2 t\,dt = \tan t + C$ [28] $\int \frac{1}{\sin^2 t}\,dt = \int \csc^2 t\,dt = -\cot t + C$

[29] $\int (\csc v \cot v \sec v)\,dv = \int \csc^2 v\,dv = -\cot v + C$

[30] $\int (4 + 4\tan^2 v)\,dv = 4\int (1 + \tan^2 v)\,dv = 4\int \sec^2 v\,dv = 4\tan v + C$

[31] $\int \frac{\sec w \sin w}{\cos w}\,dw = \int \sec w \tan w\,dw = \sec w + C$

[32] $\int \frac{\csc w \cos w}{\sin w}\,dw = \int \csc w \cot w\,dw = -\csc w + C$

[33] $\int \frac{(1 + \cot^2 z)\cot z}{\csc z}\,dz = \int \frac{\csc^2 z \cot z}{\csc z}\,dz = \int \csc z \cot z\,dz = -\csc z + C$

[34] $\int \frac{\tan z}{\cos z}\,dz = \int \tan z \sec z\,dz = \sec z + C$

[35] By Theorem (5.5)(i), $\int D_x\sqrt{x^2 + 4}\,dx = \sqrt{x^2 + 4} + C$.

[36] By Theorem (5.5)(i), $\int D_x\sqrt[3]{x^3 - 8}\,dx = \sqrt[3]{x^3 - 8} + C$.

[37] By Theorem (5.5)(i), $\int \frac{d}{dx}(\sin\sqrt[3]{x})\,dx = \sin\sqrt[3]{x} + C$.

[38] By Theorem (5.5)(i), $\int \frac{d}{dx}(\sqrt{\tan x})\,dx = \sqrt{\tan x} + C$.

[39] By Theorem (5.5)(ii), $D_x\int (x^3\sqrt{x - 4})\,dx = x^3\sqrt{x - 4}$.

[40] By Theorem (5.5)(ii), $D_x\int (x^4\sqrt[3]{x^2 + 9})\,dx = x^4\sqrt[3]{x^2 + 9}$.

[41] By Theorem (5.5)(ii), $\frac{d}{dx}\int \cot x^3\,dx = \cot x^3$.

[42] By Theorem (5.5)(ii), $\frac{d}{dx}\int \cos\sqrt{x^2 + 1}\,dx = \cos\sqrt{x^2 + 1}$.

[43] $\int a^2\,dx = a^2\int dx = a^2x + C$ [44] $\int ab\,dx = ab\int dx = abx + C$

[45] $\int (at + b)\,dt = a\cdot\frac{t^2}{2} + bt + C = \frac{1}{2}at^2 + bt + C$

[46] $\int \left(\frac{a}{b^2}t\right) dt = \frac{a}{b^2}\int t\,dt = \frac{a}{2b^2}t^2 + C$

[47] $\int (a + b)\,du = (a + b)\int du = (a + b)u + C$

[48] $\int (b - a^2)\,du = (b - a^2)\int du = (b - a^2)u + C$

[49] $f'(x) = 12x^2 - 6x + 1 \Rightarrow f(x) = 4x^3 - 3x^2 + x + C$.

$f(1) = 5 \Rightarrow 2 + C = 5 \Rightarrow C = 3$.

[50] $f'(x) = 9x^2 + x - 8 \Rightarrow f(x) = 3x^3 + \frac{1}{2}x^2 - 8x + C$.

$f(-1) = 1 \Rightarrow \frac{11}{2} + C = 1 \Rightarrow C = -\frac{9}{2}$.

[51] $\frac{dy}{dx} = 4x^{1/2} \Rightarrow y = \frac{8}{3}x^{3/2} + C$. $x = 4 \Rightarrow y = \frac{64}{3} + C = 21 \Rightarrow C = -\frac{1}{3}$.

[52] $\frac{dy}{dx} = 5x^{-1/3} \Rightarrow y = \frac{15}{2}x^{2/3} + C$. $x = 27 \Rightarrow y = \frac{135}{2} + C = 70 \Rightarrow C = \frac{5}{2}$.

[53] $f''(x) = 4x - 1 \Rightarrow f'(x) = 2x^2 - x + C$. $f'(2) = -2 \Rightarrow C = -8$.

$f(x) = x^3 - 2x^2 + x + D$. $f(2) = 4 \Rightarrow D = 2$.

[55] $\frac{d^2y}{dx^2} = 3\sin x - 4\cos x \Rightarrow \frac{dy}{dx} = -3\cos x - 4\sin x + C$. $x = 0 \Rightarrow -3 + C = 2 \Rightarrow$

$C = 5$. $y = -3\sin x + 4\cos x + 5x + D$. $x = 0 \Rightarrow 4 + D = 7 \Rightarrow D = 3$.

[56] $\frac{d^2y}{dx^2} = 2\cos x - 5\sin x \Rightarrow \frac{dy}{dx} = 2\sin x + 5\cos x + C$.

$x = \pi \Rightarrow -5 + C = 3 \Rightarrow C = 8$. $y = -2\cos x + 5\sin x + 8x + D$.

$x = \pi \Rightarrow 2 + 8\pi + D = 2 + 6\pi \Rightarrow D = -2\pi$.

[57] Since $a(t) = v'(t)$, $v(t) = 2t - 3t^2 + C$. $v(0) = -5 \Rightarrow C = -5$.

Since $v(t) = s'(t)$, $s(t) = t^2 - t^3 - 5t + D$. $s(0) = 4 \Rightarrow D = 4$.

[58] Since $a(t) = v'(t)$, $v(t) = t^3 + C$. $v(0) = 20 \Rightarrow C = 20$.

Since $v(t) = s'(t)$, $s(t) = \frac{1}{4}t^4 + 20t + D$. $s(0) = 5 \Rightarrow D = 5$.

[59] (a) $a(t) = -32 \Rightarrow v(t) = -32t + C$. $v(0) = 1600 \Rightarrow C = 1600$.

$s(t) = -16t^2 + 1600t + D$. $s(0) = 0 \Rightarrow D = 0$.

(b) The maximum height occurs when $v(t) = -32t + 1600 = 0$, or $t = 50$.

$s(50) = 40{,}000$ ft.