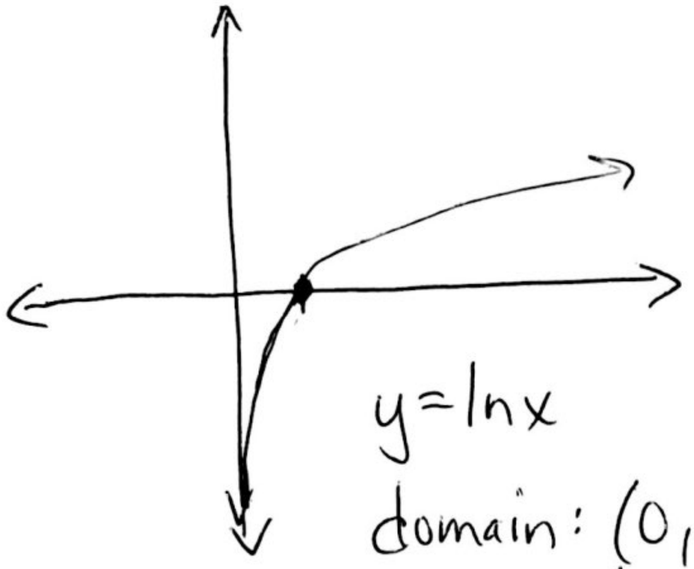
5.1 Natural Logs & Differentiation

$$\ln x = \int_1^x \frac{1}{t}\,dt \quad , \; x>0 \implies \text{Definition of } \ln(x)$$

$\ln 1 = 0$



$y = \ln x$

domain: $(0,\infty)$

range: $(-\infty,\infty)$

Vert asymptote @ $x=0$

strictly increasing

monotonic

$$\lim_{x\to 0^+}(\ln x) = -\infty$$

$$\lim_{x\to 1}(\ln x - 2) = -2$$

$$\lim_{x\to\infty}(\ln x) = \infty$$

$$\lim_{x\to\infty}\left(\ln(3x-1) - \ln(4x+7)\right)$$

$$= \lim_{x\to\infty}\left[\ln\left(\frac{3x-1}{4x+7}\right)\right] = \boxed{\ln\left(\frac{3}{4}\right)}$$

$$\lim_{x\to\infty}\left(\ln(2x+1) - \ln(3x^2+6)\right)$$

$$= \lim_{x\to\infty}\left(\ln\left(\frac{2x+1}{3x^2+6}\right)\right) = -\infty$$

Scanned by CamScanner

$$\ln x = \int_1^x \frac{1}{t}\,dt$$

$$\frac{d}{dx}\left[\ln x\right] = \frac{d}{dx}\int_1^x \frac{1}{t}\,dt = \boxed{\frac{1}{x}}\ .$$

$$\frac{d}{dx}\left[\ln |x|\right] = \frac{1}{x}$$

$y = x^3 \ln x \qquad y' = 3x^2 \ln x + x^3 \cdot \frac{1}{x} = 3x^2 \ln x + x^2 = 0$

Where is incr., decr.,
Classify local extrema.

$x^2(3\ln x + 1) = 0$

$x = 0$ $\qquad$ $3\ln x + 1 = 0$

$\ln x = -\frac{1}{3}$

$x = e^{-1/3}$

Domain $(0, \infty)$

$\uparrow$: $(e^{-1/3}, \infty)$ b/c $y' > 0$

$\downarrow$: $(0, e^{-1/3})$ b/c $y' < 0$

$-$ $\quad$ $+$

$0 \quad e^{-1/3}$

$e^{-.5}$

l. min @ $x = e^{-1/3}$ b/c y' ch $-$ to $+$

$(e^{-1/3}, -\frac{1}{3}e^{-1})$

$(e^{-1/3})^3 \cancel{\ln e^{-1/3}}$

$e^{-1}\left(-\frac{1}{3}\right)$

Remember

$\ln(e^{-1/3})$

$= -\frac{1}{3}\cancel{\ln e} = -\frac{1}{3}$

$$y = \ln(\underbrace{3x^2 - \sin x + 3})$$

$$y' = \overset{\text{deriv stuff}}{(6x - \cos x)} \cdot \frac{1}{\underset{\text{stuff}}{3x^2 - \sin x + 3}} = \boxed{\frac{6x - \cos x}{3x^2 - \sin x + 3}}$$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u} = u' \cdot \frac{1}{u} = \text{"deriv stuff} \cdot \frac{1}{\text{stuff}}$$

$$y = \ln x^2 \qquad y' = 2x \cdot \frac{1}{x^2} = \boxed{\frac{2}{x}}$$

* OR use properties

$$y = 2\ln x \qquad y' = 2 \cdot \frac{1}{x} = \boxed{\frac{2}{x}}$$

$$y = \ln x^{2} = 2\ln x$$

Same answer either way!

~~Rewrite using properties~~ 1st!

$$y = \ln\left(\frac{x^3\sqrt{x-3}}{\cos x}\right) = 3\ln x + \frac{1}{2}\ln(x-3) - \ln\cos x$$

$$y' = 3 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x-3} + \sin x \cdot \frac{1}{\cos x}$$

$$= \boxed{\frac{3}{x} + \frac{1}{2(x-3)} + \tan x}$$

Scanned by CamScanner

$$y = \ln\left(\frac{4x^4(3x-1)^3}{(x-2)^5 \sec x}\right)$$

Always rewrite using properties 1st!

$$= \ln 4 + 4\ln x + 3\ln(3x-1) - 5\ln(x-2) - \ln \sec x$$

$$y' = 4\cdot\frac{1}{x} + 3\cdot 3\cdot\frac{1}{3x-1} - 5\cdot\frac{1}{x-2} - \cancel{\sec x}\tan x\cdot\frac{1}{\cancel{\sec x}}$$

$$\boxed{y' = \frac{4}{x} + \frac{9}{3x-1} - \frac{5}{x-2} - \tan x}$$

Logarithmic Differentiation

* Take ln of both sides
* Rewrite using properties
* Take deriv of both sides

$$y = \frac{x^3\sqrt{3x^2-7}}{\sin x\,(x-7)^5}$$

Oh no! No ln! Take ln of both sides!

$$\ln y = \ln\left(\frac{x^3\sqrt{3x^2-7}}{\sin x\,(x-7)^5}\right)$$

Rewrite w/prop

$$\ln y = 3\ln x + \frac{1}{2}\ln(3x^2-7) - \ln\sin x - 5\ln(x-7)$$

Take deriv

$$y\cdot y'\cdot\frac{1}{y} = \left(3\cdot\frac{1}{x} + \frac{1}{2}\cdot(6x)\cdot\frac{1}{3x^2-7} - \cos x\cdot\frac{1}{\sin x} - 5\cdot\frac{1}{x-7}\right)y$$

Mult by y

$$\boxed{y' = \left(\frac{3}{x} + \frac{3x}{3x^2-7} - \cot x - \frac{5}{x-7}\right)\underbrace{\left(\frac{x^3\sqrt{3x^2-7}}{\sin x\,(x-7)^5}\right)}_{y}}$$

* This is when obnoxious function & don't have log but wish you did. Just take nat. log (ln) of both sides rewrite obnoxious side using properties, take deriv of both sides, solve for y' by mult by y.

Scanned by CamScanner