5.2: Integrals with Natural Logs

$$\int \frac{1}{x}\,dx = \ln|x| + c \qquad d: (-\infty, 0) \cup (0, \infty)$$

$$\int \frac{1}{u}\,du = \ln|u| + c \longleftarrow$$ u-substitutions to get this

* look for u = denominator & see if u' = numerator

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Ex. $\int \frac{1}{3x+1}\,dx$

$u = 3x+1$

$du = 3\,dx \rightarrow \frac{1}{3}\,du = dx$

$$= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|3x+1| + c = \ln\left|\sqrt[3]{3x+1}\right| + c$$

Ex. $\int \frac{x}{x^2-4}\,dx$

$u = x^2-4$

$du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|x^2-4| + c$$

$$= \ln\sqrt{|x^2-4|} + c$$

$\int \frac{\sec^2 x}{\tan x}\,dx =$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\int \frac{1}{u}\,du = \ln|u| + c = \boxed{\ln|\tan x| + c}$$

$$\int \frac{x}{\sqrt{9-x^2}}\,dx \qquad u = 9-x^2$$

$$du = -2x\,dx \Rightarrow -\frac{1}{2}du = x\,dx$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2}\cdot 2u^{1/2}+c$$

$$= \boxed{-\sqrt{9-x^2}+c}$$

OR

$$\int \frac{x}{\sqrt{9-x^2}}\,dx \qquad u = \sqrt{9-x^2}$$

$$du = \frac{-\cancel{2}x\cdot\frac{1}{\cancel{2}}}{\sqrt{9-x^2}}\,dx$$

$$-\int du = -u+c$$

$$= \boxed{-\sqrt{9-x^2}+c}$$

SAME!

$$\int \frac{x+1}{x^2+2x}\,dx \qquad u = x^2+2x$$

$$du = 2x+2\,dx = 2(x+1)\,dx$$

$$\frac{1}{2}du = (x+1)\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|+c = \boxed{\frac{1}{2}\ln|x^2+2x|+c}$$

$$\int \frac{x^2+x+1}{x^2+1}\,dx$$

If num deg ≥ denom
Always divide 1st!

$$\begin{array}{r} 1 \\ x^2+1 \,\big)\, x^2+x+1 \\ -x^2 \quad\; \mp 1 \\ \hline x \end{array}$$

$$= \int\left(1+\frac{x}{x^2+1}\right)dx$$

$$= x + \int \frac{x}{x^2+1}\,dx \qquad u = x^2+1$$

$$du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx$$

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$= x + \frac{1}{2}\ln|x^2+1|+c$$

$\int \frac{1}{x \ln x} dx$ $u = \ln x$
$du = \frac{1}{x} dx$

$= \int \frac{1}{u} du = \ln|u| + c = \boxed{\ln|\ln x| + c}$

~~$u = x \ln x$~~
~~$du = (\ln x + 1) dx$~~

KNOW THESE:

$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$ $u = \cos x$
$du = -\sin x \, dx$

$= -\int \frac{1}{u} du = -\ln|\cos x| + c = \ln|\sec x| + c$

$\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$ $u = \sin x$
$du = \cos x \, dx$

$\int \frac{1}{u} du = \ln|\sin x| + c$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\int \csc x \, dx = -\ln|\csc x + \cot x| + c = \ln|\csc x - \cot x| + c$