5.5 Other Bases

Why $y = \log_7 x = \dfrac{\ln x}{\ln 7} = \dfrac{1}{\ln 7}(\ln x)$

$y' = \dfrac{1}{\ln 7} \cdot \dfrac{1}{x} = \boxed{\dfrac{1}{x \ln 7}}$

$y = \log_a x$

$y' = \dfrac{1}{x} \cdot \dfrac{1}{\ln a} = \dfrac{1}{x \ln a}$

KNOW $\dfrac{d}{dx}[\log_a x] = \dfrac{1}{x \ln a}$

$y = \log (x^3 - 4x^2 + \sin x)$

$y' = (3x^2 - 8x + \cos x) \cdot \dfrac{1}{x^3 - 4x^2 + \sin x} \cdot \dfrac{1}{\ln 10}$

$= \dfrac{3x^2 - 8x + \cos x}{(x^3 - 4x^2 + \sin x)\ln 10}$

$y = \log_5 \left(\dfrac{x^3 \sqrt{x-1}}{\cos x} \right) = 3\log_5 x + \frac{1}{2}\log_5 (x-1) - \log_5 (\cos x)$

$y' = 3 \cdot \dfrac{1}{x \ln 5} + \dfrac{1}{2} \cdot \dfrac{1}{(x-1)\ln 5} + \sin x \cdot \dfrac{1}{(\cos x)\ln 5}$

$= \dfrac{1}{\ln 5}\left[\dfrac{3}{x} + \dfrac{1}{2(x-1)} + \tan x \right]$

WHY

$y = \ln a^x$

$\ln y = x \ln a$

$y' \cdot \frac{1}{y} = \ln a$

$y' = (\ln a) a^x$

Know: $\frac{d}{dx}[a^x] = a^x \cdot \ln a$

$\frac{d}{dx}[2^x] = 2^x \cdot \ln 2$

$\frac{d}{dx}[5^x] = 5^x \cdot \ln 5$

$\frac{d}{dx}[7^{x^2 \cos x}]$

$= (2x\cos x - x^2 \sin x) \cdot 7^{x^2 \cos x} \ln 7$

Know:

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Anytime $[f(x)]^{g(x)}$ * Must log diff *

Ex1 $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$y' \cdot \frac{1}{y} = \ln x + x \cdot \frac{1}{x}$

$y' = (\ln x + 1) x^x$

Ex2 $y = (\sin x)^{x+4}$

$\ln y = (x+4) \ln(\sin x)$

$y' \cdot \frac{1}{y} = \ln(\sin x) + (x+4)(\cos x \cdot \frac{1}{\sin x})$

$y' = (\ln(\sin x) + (x+4)\cot x)(\sin x)^{x+4}$