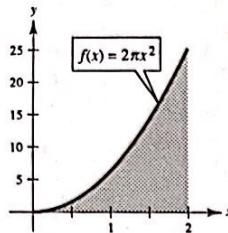


88.  $\int_0^2 2\pi x^2 dx$

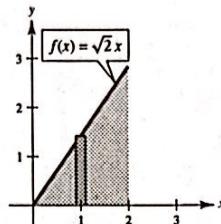
(a) Let  $f(x) = 2\pi x^2$  over the interval  $[0, 2]$ .

$$A = \int_0^2 (2\pi x^2) dx$$



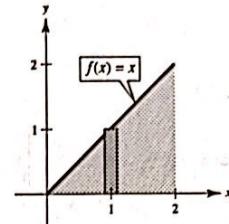
(b) Let  $f(x) = \sqrt{2}x$  over the interval  $[0, 2]$ . Revolve this region about the  $x$ -axis.

$$V = \pi \int_0^2 (\sqrt{2}x)^2 dx \\ = \int_0^2 2\pi x^2 dx$$



(c) Let  $f(x) = x$  over the interval  $[0, 2]$ . Revolve this region about the  $y$ -axis.

$$V = 2\pi \int_0^2 x(x) dx \\ = \int_0^2 2\pi x^2 dx$$



90. (a)  $\frac{1}{(\pi/n) - 0} \int_0^{\pi/n} \sin(nx) dx = \frac{1}{\pi} \int_0^{\pi/n} \sin(nx)(n) dx = \left[ -\frac{1}{\pi} \cos(nx) \right]_0^{\pi/n} = \frac{2}{\pi}$

(b)  $\frac{1}{3 - (-3)} \int_{-3}^3 \frac{1}{1 + x^2} dx = \left[ \frac{1}{6} \arctan x \right]_{-3}^3 = \frac{1}{3} \arctan 3$

92.  $y = 2\sqrt{x}$

$$y' = \frac{1}{\sqrt{x}}$$

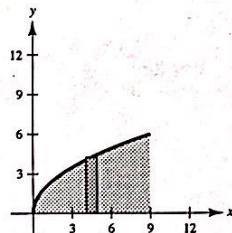
$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x+1}{x}$$

$$S = 2\pi \int_0^9 2\sqrt{x} \sqrt{\frac{x+1}{x}} dx$$

$$= 2\pi \int_0^9 2\sqrt{x+1} dx$$

$$= \left[ 4\pi \left( \frac{2}{3} \right) (x+1)^{3/2} \right]_0^9$$

$$= \frac{8\pi}{3} (10\sqrt{10} - 1) \approx 256.545$$



94.  $y = x^{2/3}$

$$y' = \frac{2}{3x^{1/3}}$$

$$1 + (y')^2 = 1 + \frac{4}{9x^{2/3}}$$

$$s = \int_1^8 \sqrt{1 + \frac{4}{9x^{2/3}}} dx \approx 7.6337$$

## Section 7.2 Integration by Parts

2.  $\frac{d}{dx} [x^2 \sin x + 2x \cos x - 2 \sin x] = x^2 \cos x + 2x \sin x - 2x \sin x + 2 \cos x - 2 \cos x = x^2 \cos x$ . Matches (d)

4.  $\frac{d}{dx}[-x + x \ln x] = -1 + x\left(\frac{1}{x}\right) + \ln x = \ln x$ . Matches (a)

6.  $\int x^2 e^{2x} dx$

$$u = x^2, dv = e^{2x} dx$$

8.  $\int \ln 3x dx$

$$u = \ln 3x, dv = dx$$

10.  $\int x^2 \cos x dx$

$$u = x^2, dv = \cos x dx$$

12.  $dv = e^{-x} dx \Rightarrow v = \int e^{-x} dx = -e^{-x}$   
 $u = x \Rightarrow du = dx$

14.  $\int \frac{e^{1/t}}{t^2} dt = - \int e^{1/t} \left(\frac{-1}{t^2}\right) dt = -e^{1/t} + C$

$$\begin{aligned} 2 \int \frac{x}{e^x} dx &= 2 \int x e^{-x} dx \\ &= 2 \left[ -xe^{-x} - \int -e^{-x} dx \right] = 2[-xe^{-x} - e^{-x}] + C \\ &= -2xe^{-x} - 2e^{-x} + C \end{aligned}$$

16.  $dv = x^4 dx \Rightarrow v = \frac{x^5}{5}$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

18. Let  $u = \ln x, du = \frac{1}{x} dx$ .

$$\int \frac{1}{x(\ln x)^3} dx = \int (\ln x)^{-3} \left(\frac{1}{x}\right) dx = \frac{-1}{2(\ln x)^2} + C$$

$$\begin{aligned} \int x^4 \ln x dx &= \frac{x^5}{5} \ln x - \int \frac{x^5}{5} \left(\frac{1}{x}\right) dx = \frac{x^5}{5} \ln x - \frac{1}{5} \int x^4 dx \\ &= \frac{x^5}{5} \ln x - \frac{1}{25} x^5 + C = \frac{x^5}{25} (5 \ln x - 1) + C \end{aligned}$$

20.  $dv = \frac{1}{x^2} dx \Rightarrow v = \int \frac{1}{x^2} dx = -\frac{1}{x}$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

22.  $dv = \frac{x}{(x^2 + 1)^2} dx \Rightarrow v = \int (x^2 + 1)^{-2} x dx = -\frac{1}{2(x^2 + 1)}$

$$u = x^2 e^{x^2} \Rightarrow du = (2x^3 e^{x^2} + 2x e^{x^2}) dx = 2x e^{x^2} (x^2 + 1) dx$$

$$\int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx = -\frac{x^2 e^{x^2}}{2(x^2 + 1)} + \int x e^{x^2} dx = -\frac{x^2 e^{x^2}}{2(x^2 + 1)} + \frac{e^{x^2}}{2} + C = \frac{e^{x^2}}{2(x^2 + 1)} + C$$

24.  $dv = \frac{1}{x^2} dx \Rightarrow v = \int \frac{1}{x^2} dx = -\frac{1}{x}$

$$u = \ln 2x \Rightarrow du = \frac{1}{x} dx$$

$$\int \frac{\ln(2x)}{x^2} dx = -\frac{\ln(2x)}{x} + \int \frac{1}{x^2} dx = -\frac{\ln(2x)}{x} - \frac{1}{x} + C = -\frac{\ln(2x) + 1}{x} + C$$

26.  $dv = \frac{1}{\sqrt{2+3x}} dx \Rightarrow v = \int (2+3x)^{-1/2} dx = \frac{2}{3}\sqrt{2+3x}$

$$u = x \quad \Rightarrow \quad du = dx$$

$$\begin{aligned} \int \frac{x}{\sqrt{2+3x}} dx &= \frac{2x\sqrt{2+3x}}{3} - \frac{2}{3} \int \sqrt{2+3x} dx \\ &= \frac{2x\sqrt{2+3x}}{3} - \frac{4}{27}(2+3x)^{3/2} + C = \frac{2\sqrt{2+3x}}{27}[9x - 2(2+3x)] + C = \frac{2\sqrt{2+3x}}{27}(3x-4) + C \end{aligned}$$

28.  $dv = \sin x dx \Rightarrow v = -\cos x$

$$u = x \quad \Rightarrow \quad du = dx$$

$$\int x \sin x dx = -x \cos x - \int -\cos x dx = -x \cos x + \sin x + C$$

30. Use integration by parts twice.

(1)  $u = x^2, du = 2x dx, dv = \cos x dx, v = \sin x$

$$\int x^2 \cos x dx = x^2 \sin x - 2 \int x \sin x dx$$

(2)  $u = x, du = dx, dv = \sin x dx, v = -\cos x$

$$\begin{aligned} \int x^2 \cos x dx &= x^2 \sin x - 2 \left[ -x \cos x + \int \cos x dx \right] \\ &= x^2 \sin x + 2x \cos x - 2 \sin x + C \end{aligned}$$

32.  $dv = \sec \theta \tan \theta d\theta \Rightarrow v = \int \sec \theta \tan \theta d\theta = \sec \theta$

$$u = \theta \quad \Rightarrow \quad du = d\theta$$

$$\begin{aligned} \int \theta \sec \theta \tan \theta d\theta &= \theta \sec \theta - \int \sec \theta d\theta \\ &= \theta \sec \theta - \ln|\sec \theta + \tan \theta| + C \end{aligned}$$

34.  $dv = dx \quad \Rightarrow \quad v = \int dx = x$

$$u = \arccos x \Rightarrow du = -\frac{1}{\sqrt{1-x^2}} dx$$

$$\begin{aligned} 4 \int \arccos x dx &= 4 \left[ x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx \right] \\ &= 4[x \arccos x - \sqrt{1-x^2}] + C \end{aligned}$$

36. Use integration by parts twice.

(1)  $dv = e^x dx \Rightarrow v = \int e^x dx = e^x$

$$u = \cos 2x \Rightarrow du = -2 \sin 2x dx$$

$$\int e^x \cos 2x dx = e^x \cos 2x + 2 \int e^x \sin 2x dx = e^x \cos 2x + 2 \left( e^x \sin 2x - 2 \int e^x \cos 2x dx \right)$$

$$5 \int e^x \cos 2x dx = e^x \cos 2x + 2e^x \sin 2x$$

$$\int e^x \cos 2x dx = \frac{e^x}{5}(\cos 2x + 2 \sin 2x) + C$$

(2)  $dv = e^x dx \Rightarrow v = \int e^x dx = e^x$

$$u = \sin 2x \Rightarrow du = 2 \cos 2x dx$$

38.  $dv = dx \Rightarrow v = x$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$y' = \ln x$$

$$y = \int \ln x dx = x \ln x - \int x \left( \frac{1}{x} \right) dx = x \ln x - x + C = x(-1 + \ln x) + C$$

40. Use integration by parts twice.

$$(1) \ dv = \sqrt{x-1} dx \Rightarrow v = \int (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}$$

$$u = x^2 \quad \Rightarrow \quad du = 2x dx$$

$$(2) \ dv = (x-1)^{3/2} dx \Rightarrow v = \int (x-1)^{3/2} dx = \frac{2}{5}(x-1)^{5/2}$$

$$u = x \quad \Rightarrow \quad du = dx$$

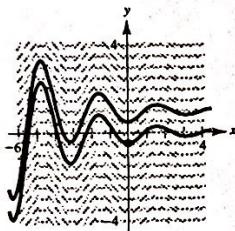
$$\begin{aligned} y &= \int x^2 \sqrt{x-1} dx \\ &= \frac{2}{3}x^2(x-1)^{3/2} - \frac{4}{3} \int x(x-1)^{3/2} dx = \frac{2}{3}x^2(x-1)^{3/2} - \frac{4}{3} \left[ \frac{2}{5}x(x-1)^{5/2} - \frac{2}{5} \int (x-1)^{5/2} dx \right] \\ &= \frac{2}{3}x^2(x-1)^{3/2} - \frac{8}{15}x(x-1)^{5/2} + \frac{16}{105}(x-1)^{7/2} + C = \frac{2(x-1)^{3/2}}{105}(15x^2 + 12x + 8) + C \end{aligned}$$

$$42. \ dv = dx \quad \Rightarrow \quad v = \int dx = x$$

$$u = \arctan \frac{x}{2} \Rightarrow du = \frac{1}{1+(x/2)^2} \left( \frac{1}{2} \right) dx = \frac{2}{4+x^2} dx$$

$$y = \int \arctan \frac{x}{2} dx = x \arctan \frac{x}{2} - \int \frac{2x}{4+x^2} dx = x \arctan \frac{x}{2} - \ln(4+x^2) + C$$

44. (a)



$$(b) \frac{dy}{dx} = e^{-x/3} \sin 2x, \left( 0, -\frac{18}{37} \right)$$

$$y = \int e^{-x/3} \sin 2x dx$$

Use integration by parts twice.

$$(1) \ u = \sin 2x, \ du = 2 \cos 2x$$

$$dv = e^{-x/3} dx, v = -3e^{-x/3}$$

$$\int e^{-x/3} \sin 2x dx = -3e^{-x/3} \sin 2x + \int 6e^{-x/3} \cos 2x dx$$

$$(2) \ u = \cos 2x, \ du = -2 \sin 2x$$

$$dv = e^{-x/3} dx, v = -3e^{-x/3}$$

$$\int e^{-x/3} \sin 2x dx = -3e^{-x/3} \sin 2x + 6 \left[ -3e^{-x/3} \cos 2x - \int 6e^{-x/3} \sin 2x dx \right] + C$$

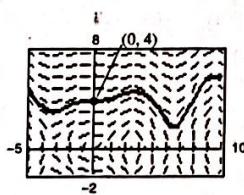
$$37 \int e^{-x/3} \sin 2x dx = -3e^{-x/3} \sin 2x - 18e^{-x/3} \cos 2x + C$$

$$y = \int e^{-x/3} \sin 2x dx = \frac{1}{37} \left[ -3e^{-x/3} \sin 2x - 18e^{-x/3} \cos 2x \right] + C$$

$$\left( 0, \frac{-18}{37} \right): \frac{-18}{37} = \frac{1}{37}[0 - 18] + C \Rightarrow C = 0$$

$$y = \frac{-1}{37} [3e^{-x/3} \sin 2x + 18e^{-x/3} \cos 2x]$$

46.  $\frac{dy}{dx} = \frac{x}{y} \sin x, y(0) = 4$



48. See Exercise 3.

$$\int_0^1 x^2 e^x dx = \left[ x^2 e^x - 2xe^x + 2e^x \right]_0^1 = e - 2 \approx 0.718$$

50.  $dv = \sin 2x dx \Rightarrow v = \int \sin 2x dx = -\frac{1}{2} \cos 2x$

$$u = x \Rightarrow du = dx$$

$$\begin{aligned} \int x \sin 2x dx &= \frac{-1}{2} x \cos 2x + \frac{1}{2} \int \cos 2x dx \\ &= \frac{-1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C \\ &= \frac{1}{4} (\sin 2x - 2x \cos 2x) + C \end{aligned}$$

Thus,  $\int_0^\pi x \sin 2x dx = \left[ \frac{1}{4} (\sin 2x - 2x \cos 2x) \right]_0^\pi = -\frac{\pi}{2}$ .

52.  $dv = x dx \Rightarrow v = \int x dx = \frac{x^2}{2}$

$$u = \arcsin x^2 \Rightarrow du = \frac{2x}{\sqrt{1-x^4}} dx$$

$$\begin{aligned} \int x \arcsin x^2 dx &= \frac{x^2}{2} \arcsin x^2 - \int \frac{x^3}{\sqrt{1-x^4}} dx \\ &= \frac{x^2}{2} \arcsin x^2 + \frac{1}{4} (2)(1-x^4)^{1/2} + C \\ &= \frac{1}{2} [x^2 \arcsin x^2 + \sqrt{1-x^4}] + C \end{aligned}$$

Thus,  $\int_0^1 x \arcsin x^2 dx = \frac{1}{2} \left[ x^2 \arcsin x^2 + \sqrt{1-x^4} \right]_0^1 = \frac{1}{4} (\pi - 2)$ .

54. Use integration by parts twice.

(1)  $dv = e^{-x}, v = -e^{-x}, u = \cos x, du = -\sin x dx$

$$\int e^{-x} \cos x dx = -e^{-x} \cos x - \int e^{-x} \sin x dx$$

(2)  $dv = e^{-x} dx, v = -e^{-x}, u = \sin x, du = \cos x dx$

$$\int e^{-x} \cos x dx = -e^{-x} \cos x - \left[ -e^{-x} \sin x + \int e^{-x} \cos x dx \right] \Rightarrow 2 \int e^{-x} \cos x dx = e^{-x} \sin x - e^{-x} \cos x$$

Thus,

$$\begin{aligned} \int_0^2 e^{-x} \cos x dx &= \left[ \frac{e^{-x} \sin x - e^{-x} \cos x}{2} \right]_0^2 \\ &= \frac{-e^{-2}}{2} [\sin 2 - \cos 2] + \frac{1}{2} \end{aligned}$$

56.  $dv = dx \Rightarrow v = \int dx = x$

$$u = \ln(1+x^2) \Rightarrow du = \frac{2x}{1+x^2} dx$$

$$\begin{aligned} \int \ln(1+x^2) dx &= x \ln(1+x^2) - \int \frac{2x^2}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \left[ 1 - \frac{1}{1+x^2} \right] dx = x \ln(1+x^2) - 2x + 2 \arctan x + C \end{aligned}$$

Thus,  $\int_0^1 \ln(1+x^2) dx = \left[ x \ln(1+x^2) - 2x + 2 \arctan x \right]_0^1 = \ln 2 - 2 + \frac{\pi}{2}$ .

58.  $u = x, du = dx, dv = \sec^2 x dx, v = \tan x$

$$\int x \sec^2 x dx = x \tan x - \int \tan x dx$$

Hence,

$$\begin{aligned} \int_0^{\pi/4} x \sec^2 x dx &= \left[ x \tan x + \ln |\cos x| \right]_0^{\pi/4} \\ &= \left( \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2} \right) - 0 \\ &= \frac{\pi}{4} - \frac{1}{2} \ln 2 \end{aligned}$$

60.  $\int x^3 e^{-2x} dx = x^3 \left( -\frac{1}{2} e^{-2x} \right) - 3x^2 \left( \frac{1}{4} e^{-2x} \right) + 6x \left( -\frac{1}{8} e^{-2x} \right) - 6 \left( \frac{1}{16} e^{-2x} \right) + C$   
 $= -\frac{1}{8} e^{-2x} (4x^3 + 6x^2 + 6x + 3) + C$

Alternate signs	$u$ and its derivatives	$v'$ and its antiderivatives
+	$x^3$	$e^{-2x}$
-	$3x^2$	$-\frac{1}{2}e^{-2x}$
+	$6x$	$\frac{1}{4}e^{-2x}$
-	$6$	$-\frac{1}{8}e^{-2x}$
+	$0$	$\frac{1}{16}e^{-2x}$

62.  $\int x^3 \cos 2x dx = x^3 \left( \frac{1}{2} \sin 2x \right) - 3x^2 \left( -\frac{1}{4} \cos 2x \right) + 6x \left( -\frac{1}{8} \sin 2x \right) - 6 \left( \frac{1}{16} \cos 2x \right) + C$   
 $= \frac{1}{2} x^3 \sin 2x + \frac{3}{4} x^2 \cos 2x - \frac{3}{4} x \sin 2x - \frac{3}{8} \cos 2x + C$   
 $= \frac{1}{8} [4x^3 \sin 2x + 6x^2 \cos 2x - 6x \sin 2x - 3 \cos 2x] + C$

Alternate signs	$u$ and its derivatives	$v'$ and its antiderivatives
+	$x^3$	$\cos 2x$
-	$3x^2$	$\frac{1}{2} \sin 2x$
+	$6x$	$-\frac{1}{4} \cos 2x$
-	$6$	$-\frac{1}{8} \sin 2x$
+	$0$	$\frac{1}{16} \cos 2x$

64.  $\int x^2(x-2)^{3/2} dx = \frac{2}{5} x^2(x-2)^{5/2} - \frac{8}{35} x(x-2)^{7/2} + \frac{16}{315} (x-2)^{9/2} + C$   
 $= \frac{2}{315} (x-2)^{5/2} (35x^2 + 40x + 32) + C$

Alternate signs	$u$ and its derivatives	$v'$ and its antiderivatives
+	$x^2$	$(x-2)^{3/2}$
-	$2x$	$\frac{2}{5} (x-2)^{5/2}$
+	$2$	$\frac{4}{35} (x-2)^{7/2}$
-	$0$	$\frac{8}{315} (x-2)^{9/2}$

66. Answers will vary.  
See pages 488, 493.

68. Yes.  
 $u = \ln x, dv = x dx$

70. No. Substitution.

72. No. Substitution.

74.  $\int \alpha^4 \sin \pi \alpha d\alpha = \frac{1}{\pi^5} [-(\alpha \pi)^4 \cos \pi \alpha + 4(\alpha \pi)^3 \sin \pi \alpha + 12(\alpha \pi)^2 \cos \pi \alpha - 24(\alpha \pi) \sin \pi \alpha - 24 \cos \pi \alpha] + C$

76.  $\int_0^5 x^4 (25-x^2)^{3/2} dx = \left[ \frac{1,171,875 \arcsin(x/5)}{128} - \frac{x(2x^2+25)(25-x^2)^{5/2}}{16} + \frac{625x(25-x^2)^{3/2}}{64} + \frac{46,875x\sqrt{25-x^2}}{128} \right]_0^5$   
 $\approx 14,381.0699$

78. (a)  $dv = \sqrt{4+x} dx \Rightarrow v = \int (4+x)^{1/2} dx = \frac{2}{3}(4+x)^{3/2}$

$$u = x \quad \Rightarrow \quad du = dx$$

$$\begin{aligned} \int x\sqrt{4+x} dx &= \frac{2}{3}x(4+x)^{3/2} - \frac{2}{3}\int (4+x)^{3/2} dx \\ &= \frac{2}{3}x(4+x)^{3/2} - \frac{4}{15}(4+x)^{5/2} + C = \frac{2}{15}(4+x)^{3/2}(3x-8) + C \end{aligned}$$

(b)  $u = 4+x \Rightarrow x = u-4$  and  $dx = du$

$$\begin{aligned} \int x\sqrt{4+x} dx &= \int (u-4)u^{1/2}du = \int (u^{3/2} - 4u^{1/2}) du \\ &= \frac{2}{5}u^{5/2} - \frac{8}{3}u^{3/2} + C = \frac{2}{15}u^{3/2}(3u-20) + C \\ &= \frac{2}{15}(4+x)^{3/2}[3(4+x)-20] + C = \frac{2}{15}(4+x)^{3/2}(3x-8) + C \end{aligned}$$

80. (a)  $dv = \sqrt{4-x} dx \Rightarrow v = \int (4-x)^{1/2} dx$

$$\begin{aligned} &= -\frac{2}{3}(4-x)^{3/2} \\ u = x &\quad \Rightarrow \quad du = dx \\ \int x\sqrt{4-x} dx &= -\frac{2}{3}x(4-x)^{3/2} + \frac{2}{3}\int (4-x)^{3/2} dx \\ &= -\frac{2}{3}x(4-x)^{3/2} - \frac{4}{15}(4-x)^{5/2} + C \\ &= -\frac{2}{15}(4-x)^{3/2}[5x+2(4-x)] + C \\ &= -\frac{2}{15}(4-x)^{3/2}(3x+8) + C \end{aligned}$$

(b)  $u = 4-x \Rightarrow x = 4-u$  and  $dx = -du$

$$\begin{aligned} \int x\sqrt{4-x} dx &= -\int (4-u)\sqrt{u} du \\ &= -\int (4u^{1/2} - u^{3/2}) du \\ &= -\frac{8}{3}u^{3/2} + \frac{2}{5}u^{5/2} + C \\ &= -\frac{2}{15}u^{3/2}(20-3u) + C \\ &= -\frac{2}{15}(4-x)^{3/2}[20-3(4-x)] + C \\ &= -\frac{2}{15}(4-x)^{3/2}(3x+8) + C \end{aligned}$$

84.  $dv = \cos x dx \Rightarrow v = \sin x$

$$u = x^n \quad \Rightarrow \quad du = nx^{n-1} dx$$

$$\int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$$

82.  $n = 0: \int e^x dx = e^x + C$

$$\begin{aligned} n = 1: \int xe^x dx &= xe^x - e^x + C = xe^x - \int e^x dx \\ n = 2: \int x^2 e^x dx &= x^2 e^x - 2xe^x + 2e^x + C \\ &= x^2 e^x - 2 \int xe^x dx \end{aligned}$$

$$\begin{aligned} n = 3: \int x^3 e^x dx &= x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C \\ &= x^3 e^x - 3 \int x^2 e^x dx \end{aligned}$$

$$\begin{aligned} n = 4: \int x^4 e^x dx &= x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24xe^x + 24e^x + C \\ &= x^4 e^x - 4 \int x^3 e^x dx \end{aligned}$$

In general,  $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$ .

(See Exercise 86)

86.  $dv = e^{ax} dx \Rightarrow v = \frac{1}{a}e^{ax}$

$$u = x^n \Rightarrow du = nx^{n-1} dx$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

88. Use integration by parts twice.

$$(1) \ dv = e^{ax} dx \Rightarrow v = \frac{1}{a} e^{ax}$$

$$u = \cos bx \Rightarrow du = -b \sin bx$$

$$(2) \ dv = e^{ax} dx \Rightarrow v = \frac{1}{a} e^{ax}$$

$$u = \sin bx \Rightarrow du = b \cos bx$$

$$\begin{aligned} \int e^{ax} \cos bx dx &= \frac{e^{ax} \cos bx}{a} + \frac{b}{a} \int e^{ax} \sin bx dx = \frac{e^{ax} \cos bx}{a} + \frac{b}{a} \left[ \frac{e^{ax} \sin bx}{a} - \frac{b}{a} \int e^{ax} \cos bx dx \right] \\ &= \frac{e^{ax} \cos bx}{a} + \frac{be^{ax} \sin bx}{a^2} - \frac{b^2}{a^2} \int e^{ax} \cos bx dx \end{aligned}$$

$$\text{Therefore, } \left(1 + \frac{b^2}{a^2}\right) \int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2}$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C.$$

90.  $n = 2$  (Use formula in Exercise 84.)

$$\begin{aligned} \int x^2 \cos x dx &= x^2 \sin x - 2 \int x \sin x dx \quad (\text{Use formula in Exercise 83.}) \quad (n = 1) \\ &= x^2 \sin x - 2 \left[ -x \cos x + \int \cos x dx \right] = x^2 \sin x + 2x \cos x - 2 \sin x + C \end{aligned}$$

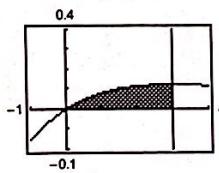
92.  $n = 3, a = 2$  (Use formula in Exercise 86 three times.)

$$\begin{aligned} \int x^3 e^{2x} dx &= \frac{x^3 e^{2x}}{2} - \frac{3}{2} \int x^2 e^{2x} dx \quad (n = 3, a = 2) \\ &= \frac{x^3 e^{2x}}{2} - \frac{3}{2} \left[ \frac{x^2 e^{2x}}{2} - \int x e^{2x} dx \right] \quad (n = 2, a = 2) \\ &= \frac{x^3 e^{2x}}{2} - \frac{3x^2 e^{2x}}{4} + \frac{3}{2} \left[ \frac{x e^{2x}}{2} - \frac{1}{2} \int e^{2x} dx \right] = \frac{x^3 e^{2x}}{2} - \frac{3x^2 e^{2x}}{4} + \frac{3x e^{2x}}{4} - \frac{3e^{2x}}{8} + C \quad (n = 1, a = 2) \\ &= \frac{e^{2x}}{8} (4x^3 - 6x^2 + 6x - 3) + C \end{aligned}$$

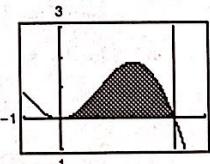
94.  $dv = e^{-x/3} dx \Rightarrow v = -3e^{-x/3}$

$$u = x \Rightarrow du = dx$$

$$\begin{aligned} A &= \frac{1}{9} \int_0^3 x e^{-x/3} dx \\ &= \frac{1}{9} \left( \left[ -3x e^{-x/3} \right]_0^3 + 3 \int_0^3 e^{-x/3} dx \right) \\ &= \frac{1}{9} \left( \frac{-9}{e} - \left[ 9e^{-x/3} \right]_0^3 \right) \\ &= -\frac{1}{e} - \frac{1}{e} + 1 = 1 - \frac{2}{e} \approx 0.264 \end{aligned}$$



$$\begin{aligned} 96. A &= \int_0^\pi x \sin x dx = \left[ -x \cos x + \sin x \right]_0^\pi \\ &= \pi \quad (\text{See Exercise 83.}) \end{aligned}$$



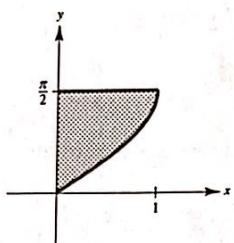
98. In Example 6, we showed that the centroid of an equivalent region was  $(1, \pi/8)$ . By symmetry, the centroid of this region is  $(\pi/8, 1)$ .

You can also solve this problem directly.

$$\begin{aligned} A &= \int_0^1 \left( \frac{\pi}{2} - \arcsin x \right) dx = \left[ \frac{\pi}{2}x - x \arcsin x - \sqrt{1-x^2} \right]_0^1 \text{ (Example 3)} \\ &= \left( \frac{\pi}{2} - \frac{\pi}{2} - 0 \right) - (-1) = 1 \end{aligned}$$

$$\bar{x} = \frac{M_y}{A} = \int_0^1 x \left[ \frac{\pi}{2} - \arcsin x \right] dx = \frac{\pi}{8}$$

$$\bar{y} = \frac{M_x}{A} = \int_0^1 \frac{(\pi/2) + \arcsin x}{2} \left[ \frac{\pi}{2} - \arcsin x \right] dx = 1$$



100. (a) Average =  $\int_1^2 (1.6t \ln t + 1) dt = \left[ 0.8t^2 \ln t - 0.4t^2 + t \right]_1^2 = 3.2(\ln 2) - 0.2 \approx 2.018$

(b) Average =  $\int_3^4 (1.6t \ln t + 1) dt = \left[ 0.8t^2 \ln t - 0.4t^2 + t \right]_3^4 = 12.8(\ln 4) - 7.2(\ln 3) - 1.8 \approx 8.035$

102.  $c(t) = 30,000 + 500t$ ,  $r = 7\%$ ,  $t_1 = 5$

$$P \int_0^5 (30,000 + 500t)e^{-0.07t} dt = 500 \int_0^5 (60 + t)e^{-0.07t} dt$$

Let  $u = 60 + t$ ,  $dv = e^{-0.07t} dt$ ,  $du = dt$ ,  $v = -\frac{100}{7}e^{-0.07t}$ .

$$\begin{aligned} P &= 500 \left\{ \left[ (60 + t) \left( -\frac{100}{7}e^{-0.07t} \right) \right]_0^5 + \frac{100}{7} \int_0^5 e^{-0.07t} dt \right\} \\ &= 500 \left\{ \left[ (60 + t) \left( -\frac{100}{7}e^{-0.07t} \right) \right]_0^5 - \left[ \frac{10,000}{49}e^{-0.07t} \right]_0^5 \right\} \approx \$131,528.68 \end{aligned}$$

104.  $\int_{-\pi}^{\pi} x^2 \cos nx dx = \left[ \frac{x^2}{n} \sin nx + \frac{2x}{n^2} \cos nx - \frac{2}{n^3} \sin nx \right]_{-\pi}^{\pi}$

$$= \frac{2\pi}{n^2} \cos n\pi + \frac{2\pi}{n^2} \cos(-n\pi)$$

$$= \frac{4\pi}{n^2} \cos n\pi$$

$$= \begin{cases} (4\pi/n^2), & \text{if } n \text{ is even} \\ -(4\pi/n^2), & \text{if } n \text{ is odd} \end{cases}$$

$$= \frac{(-1)^n 4\pi}{n^2}$$

106. For any integrable function,  $\int f(x) dx = C + \int f(x) dx$ , but this cannot be used to imply that  $C = 0$ .

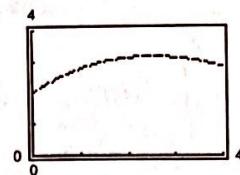
108. On  $\left[0, \frac{\pi}{2}\right]$ ,  $\sin x \leq 1 \Rightarrow x \sin x \leq x \Rightarrow \int_0^{\pi/2} x \sin x dx \leq \int_0^{\pi/2} x dx$ .

110.  $f'(x) = \cos \sqrt{x}$ ,  $f(0) = 2$

(a) It cannot be solved by integration.

(b) You obtain the points

$n$	$x_n$	$y_n$
0	0	2
1	0.05	2.05
2	0.10	2.098755
3	0.15	2.146276
$\vdots$	$\vdots$	$\vdots$
80	4.0	2.8403565



## Section 7.3 Trigonometric Integrals

2. (a)  $y = \sec x \Rightarrow y' = \sec x \tan x = \sin x \sec^2 x$ .

Matches (iii)

(b)  $y = \cos x + \sec x \Rightarrow y' = -\sin x + \sec x \tan x$

$= -\sin x + \sec^2 x \sin x$

$= \sin x(-1 + \sec^2 x)$

$= \sin x \tan^2 x$  Matches (i)

(c)  $y = x - \tan x + \frac{1}{3} \tan^3 x \Rightarrow y' = 1 - \sec^2 x + \tan^2 x \sec^2 x$   
 $= -\tan^2 x + \tan^2 x(1 + \tan^2 x)$   
 $= \tan^4 x$  Matches (iv)

(d)  $y = 3x + 2 \sin x \cos^3 x + 3 \sin x \cos x \Rightarrow$

$$\begin{aligned}y' &= 3 + 2 \cos x(\cos^3 x) + 6 \sin x \cos^2 x(-\sin x) + 3 \cos^2 x - 3 \sin^2 x \\&= 3 + 2 \cos^4 x - 6 \cos^2 x(1 - \cos^2 x) + 3 \cos^2 x - 3(1 - \cos^2 x) \\&= 8 \cos^4 x\end{aligned}$$

Matches (ii)

4.  $\int \cos^3 x \sin^4 x \, dx = \int \cos x(1 - \sin^2 x)\sin^4 x \, dx$   
 $= \int (\sin^4 x - \sin^6 x)\cos x \, dx$   
 $= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$

6. Let  $u = \cos x$ ,  $du = -\sin x \, dx$ .

$$\begin{aligned}\int \sin^3 x \, dx &= \int \sin x(1 - \cos^2 x) \, dx \\&= \int \cos^2 x(-\sin x) \, dx + \int \sin x \, dx \\&= \frac{1}{3} \cos^3 x - \cos x + C\end{aligned}$$

8. Let  $u = \sin \frac{x}{3}$ ,  $du = \frac{1}{3} \cos \frac{x}{3} \, dx$ .

$$\begin{aligned}\int \cos^3 \frac{x}{3} \, dx &= \int \left(\cos \frac{x}{3}\right) \left(1 - \sin^2 \frac{x}{3}\right) \, dx \\&= 3 \int \left(1 - \sin^2 \frac{x}{3}\right) \left(\frac{1}{3} \cos \frac{x}{3}\right) \, dx \\&= 3 \left(\sin \frac{x}{3} - \frac{1}{3} \sin^3 \frac{x}{3}\right) + C \\&= 3 \sin \frac{x}{3} - \sin^3 \frac{x}{3} + C\end{aligned}$$