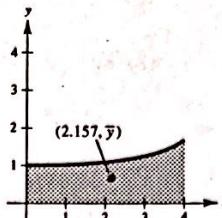


$$\begin{aligned}
 91. A &= \int_0^4 \frac{5}{\sqrt{25 - x^2}} dx = \left[5 \arcsin \frac{x}{5} \right]_0^4 = 5 \arcsin \frac{4}{5} \\
 \bar{x} &= \frac{1}{A} \int_0^4 x \left(\frac{5}{\sqrt{25 - x^2}} \right) dx \\
 &= \frac{1}{5 \arcsin(4/5)} \left(-\frac{5}{2} \right) \int_0^4 (25 - x^2)^{-1/2} (-2x) dx \\
 &= \frac{1}{5 \arcsin(4/5)} (-5) \left[(25 - x^2)^{1/2} \right]_0^4 \\
 &= -\frac{1}{\arcsin(4/5)} [3 - 5] \\
 &= \frac{2}{\arcsin(4/5)} \approx 2.157
 \end{aligned}$$



$$93. y = \tan(\pi x)$$

$$y' = \pi \sec^2(\pi x)$$

$$1 + (y')^2 = 1 + \pi^2 \sec^4(\pi x)$$

$$\begin{aligned}
 s &= \int_0^{1/4} \sqrt{1 + \pi^2 \sec^4(\pi x)} dx \\
 &\approx 1.0320
 \end{aligned}$$

Section 7.2 Integration by Parts

$$1. \frac{d}{dx} [\sin x - x \cos x] = \cos x - (-x \sin x + \cos x) = x \sin x. \text{ Matches (b)}$$

$$3. \frac{d}{dx} [x^2 e^x - 2xe^x + 2e^x] = x^2 e^x + 2xe^x - 2xe^x - 2e^x + 2e^x = x^2 e^x. \text{ Matches (c)}$$

$$\begin{aligned}
 5. \int xe^{2x} dx \\
 u = x, dv = e^{2x} dx
 \end{aligned}$$

$$\begin{aligned}
 7. \int (\ln x)^2 dx \\
 u = (\ln x)^2, dv = dx
 \end{aligned}$$

$$\begin{aligned}
 9. \int x \sec^2 x dx \\
 u = x, dv = \sec^2 x dx
 \end{aligned}$$

$$\begin{aligned}
 11. dv = e^{-2x} dx \Rightarrow v = \int e^{-2x} dx = -\frac{1}{2} e^{-2x} \\
 u = x \quad \Rightarrow du = dx
 \end{aligned}$$

$$\begin{aligned}
 \int xe^{-2x} dx &= -\frac{1}{2} xe^{-2x} - \int -\frac{1}{2} e^{-2x} dx \\
 &= -\frac{1}{2} xe^{-2x} - \frac{1}{4} e^{-2x} + C = \frac{-1}{4e^{2x}} (2x + 1) + C
 \end{aligned}$$

13. Use integration by parts three times.

$$(1) dv = e^x dx \Rightarrow v = \int e^x dx = e^x \quad (2) dv = e^x dx \Rightarrow v = \int e^x dx = e^x \quad (3) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = x^3 \Rightarrow du = 3x^2 dx \quad u = x^2 \Rightarrow du = 2x dx \quad u = x \Rightarrow du = dx$$

$$\begin{aligned} \int x^3 e^x dx &= x^3 e^x - 3 \int x^2 e^x dx = x^3 e^x - 3x^2 e^x + 6 \int x e^x dx \\ &= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C = e^x(x^3 - 3x^2 + 6x - 6) + C \end{aligned}$$

$$15. \int x^2 e^{x^3} dx = \frac{1}{3} \int e^{x^3} (3x^2) dx = \frac{1}{3} e^{x^3} + C$$

$$17. dv = t dt \Rightarrow v = \int t dt = \frac{t^2}{2}$$

$$u = \ln(t+1) \Rightarrow du = \frac{1}{t+1} dt$$

$$\begin{aligned} \int t \ln(t+1) dt &= \frac{t^2}{2} \ln(t+1) - \frac{1}{2} \int \frac{t^2}{t+1} dt \quad \text{long divide} \\ &= \frac{t^2}{2} \ln(t+1) - \frac{1}{2} \int \left(t - 1 + \frac{1}{t+1} \right) dt \\ &= \frac{t^2}{2} \ln(t+1) - \frac{1}{2} \left[\frac{t^2}{2} - t + \ln(t+1) \right] + C \\ &= \frac{1}{4} [2(t^2 - 1) \ln|t+1| - t^2 + 2t] + C \end{aligned}$$

$$19. \text{Let } u = \ln x, du = \frac{1}{x} dx.$$

$$\int \frac{(\ln x)^2}{x} dx = \int (\ln x)^2 \left(\frac{1}{x} \right) dx = \frac{(\ln x)^3}{3} + C$$

$$21. dv = \frac{1}{(2x+1)^2} dx \Rightarrow v = \int (2x+1)^{-2} dx$$

$$= -\frac{1}{2(2x+1)}$$

$$\begin{aligned} u &= xe^{2x} \quad \Rightarrow du = (2xe^{2x} + e^{2x}) dx \\ &\Rightarrow du = e^{2x}(2x+1) dx \end{aligned}$$

$$\int \frac{xe^{2x}}{(2x+1)^2} dx = -\frac{xe^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{2} dx$$

$$= \frac{-xe^{2x}}{2(2x+1)} + \frac{e^{2x}}{4} + C$$

$$= \frac{e^{2x}}{4(2x+1)} + C$$

23. Use integration by parts twice.

$$(1) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = x^2 \Rightarrow du = 2x dx$$

$$\int (x^2 - 1)e^x dx = \int x^2 e^x dx - \int e^x dx = x^2 e^x - 2 \int x e^x dx - e^x$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right] - e^x = x^2 e^x - 2x e^x + e^x + C = (x-1)^2 e^x + C$$

$$(2) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = x \Rightarrow du = dx$$

$$25. dv = \sqrt{x-1} dx \Rightarrow v = \int (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$\begin{aligned}\int x\sqrt{x-1} dx &= \frac{2}{3}x(x-1)^{3/2} - \frac{2}{3} \int (x-1)^{3/2} dx \\ &= \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C \\ &= \frac{2(x-1)^{3/2}}{15}(3x+2) + C\end{aligned}$$

$$27. dv = \cos x dx \Rightarrow v = \int \cos x dx = \sin x$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

29. Use integration by parts three times.

$$(1) u = x^3, du = 3x^2, dv = \sin x dx, v = -\cos x$$

$$\int x^3 \sin x dx = -x^3 \cos x + 3 \int x^2 \cos x dx$$

$$(2) u = x^2, du = 2x dx, dv = \cos x dx, v = \sin x$$

$$\begin{aligned}\int x^3 \sin x dx &= -x^3 \cos x + 3 \left[x^2 \sin x - 2 \int x \sin x dx \right] \\ &= -x^3 \cos x + 3x^2 \sin x - 6 \int x \sin x dx\end{aligned}$$

$$(3) u = x, du = dx, dv = \sin x dx, v = -\cos x$$

$$\begin{aligned}\int x^3 \sin x dx &= -x^3 \cos x + 3x^2 \sin x - 6 \left[-x \cos x + \int \cos x dx \right] \\ &= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C\end{aligned}$$

$$31. u = t, du = dt, dv = \csc t \cot t dt, v = -\csc t$$

$$\begin{aligned}\int t \csc t \cot t dt &= -t \csc t + \int \csc t dt \\ &= -t \csc t - \ln|\csc t + \cot t| + C\end{aligned}$$

$$33. dv = dx \quad \Rightarrow \quad v = \int dx = x$$

$$u = \arctan x \Rightarrow du = \frac{1}{1+x^2} dx$$

$$\int \arctan x dx = x \arctan x - \int \frac{x}{1+x^2} dx$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

35. Use integration by parts twice.

$$(1) dv = e^{2x} dx \Rightarrow v = \int e^{2x} dx = \frac{1}{2}e^{2x}$$

$$u = \sin x \Rightarrow du = \cos x dx$$

$$\int e^{2x} \sin x dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{2} \int e^{2x} \cos x dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{2} \left(\frac{1}{2}e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x dx \right)$$

$$\frac{5}{4} \int e^{2x} \sin x dx = \frac{1}{2}e^{2x} \sin x - \frac{1}{4}e^{2x} \cos x$$

$$\int e^{2x} \sin x dx = \frac{1}{5}e^{2x}(2 \sin x - \cos x) + C$$

$$(2) dv = e^{2x} dx \Rightarrow v = \int e^{2x} dx = \frac{1}{2}e^{2x}$$

$$u = \cos x \Rightarrow du = -\sin x dx$$

37. $y' = xe^{x^2}$

$$y = \int xe^{x^2} dx = \frac{1}{2}e^{x^2} + C$$

39. Use integration by parts twice.

$$(1) dv = \frac{1}{\sqrt{2+3t}} dt \Rightarrow v = \int (2+3t)^{-1/2} dt = \frac{2}{3}\sqrt{2+3t}$$

$$u = t^2 \Rightarrow du = 2t dt$$

$$(2) dv = \sqrt{2+3t} dt \Rightarrow v = \int (2+3t)^{1/2} dt = \frac{2}{9}(2+3t)^{3/2}$$

$$u = t \Rightarrow du = dt$$

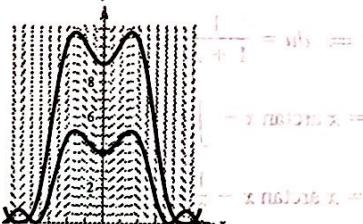
$$\begin{aligned} y &= \int \frac{t^2}{\sqrt{2+3t}} dt = \frac{2t^2\sqrt{2+3t}}{3} - \frac{4}{3} \int t\sqrt{2+3t} dt \\ &= \frac{2t^2\sqrt{2+3t}}{3} - \frac{4}{3} \left[\frac{2t}{9}(2+3t)^{3/2} - \frac{2}{9} \int (2+3t)^{3/2} dt \right] \\ &= \frac{2t^2\sqrt{2+3t}}{3} - \frac{8t}{27}(2+3t)^{3/2} + \frac{16}{405}(2+3t)^{5/2} + C \\ &= \frac{2\sqrt{2+3t}}{405} (27t^2 - 24t + 32) + C \end{aligned}$$

41. $(\cos y)y' = 2x$

$$\int \cos y dy = \int 2x dx$$

$$\sin y = x^2 + C$$

43. (a) $x = y^2$ $\Rightarrow y = \sqrt{x}$ $\Rightarrow y' = \frac{1}{2\sqrt{x}}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

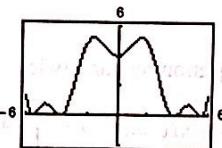


(b) $\frac{dy}{dx} = x\sqrt{y} \cos x, (0, 4)$

$$\begin{aligned} \int \frac{dy}{\sqrt{y}} &= \int x \cos x dx \\ \int y^{-1/2} dy &= \int x \cos x dx \quad (u = x, du = dx, dv = \cos x dx, v = \sin x) \end{aligned}$$

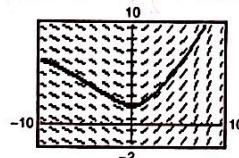
$$\begin{aligned} 2y^{1/2} &= x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

$$2\sqrt{y} = x \sin x + \cos x + 3$$



45. $\frac{dy}{dx} = \frac{x}{y} e^{x/8}, y(0) = 2$

$$\begin{aligned} \left(\ln y + \frac{1}{8} \ln x^2 + C_1 \right) \frac{1}{x} &= x \ln x^2 + C_2 \\ \ln y + \frac{1}{8} \ln x^2 &= x \ln x^2 + C_2 \\ x \ln x^2 + C_2 &= x \ln x^2 + C_3 \end{aligned}$$



47. $u = x, du = dx, dv = e^{-x/2} dx, v = -2e^{-x/2}$

$$\int xe^{-x/2} dx = -2xe^{-x/2} + \int 2e^{-x/2} dx = -2xe^{-x/2} - 4e^{-x/2} + C$$

Thus, $\int_0^4 xe^{-x/2} dx = \left[-2xe^{-x/2} - 4e^{-x/2} \right]_0^4$
 $= -8e^{-2} - 4e^{-2} + 4$
 $= -12e^{-2} + 4 \approx 2.376.$

49. See Exercise 27.

$$\int_0^{\pi/2} x \cos x dx = \left[x \sin x + \cos x \right]_0^{\pi/2} = \frac{\pi}{2} - 1$$

51. $u = \arccos x, du = -\frac{1}{\sqrt{1-x^2}} dx, dv = dx, v = x$

$$\int \arccos x dx = x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx = x \arccos x - \sqrt{1-x^2} + C$$

Thus, $\int_0^{1/2} \arccos x = \left[x \arccos x - \sqrt{1-x^2} \right]_0^{1/2}$
 $= \frac{1}{2} \arccos\left(\frac{1}{2}\right) - \sqrt{\frac{3}{4}} + 1$
 $= \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1 \approx 0.658.$

53. Use integration by parts twice.

(1) $dv = e^x dx \Rightarrow v = \int e^x dx = e^x$

$u = \sin x \Rightarrow du = \cos x dx$

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx = e^x \sin x - e^x \cos x - \int e^x \sin x dx$$

$$2 \int e^x \sin x dx = e^x (\sin x - \cos x)$$

$$\int e^x \sin x dx = \frac{e^x}{2} (\sin x - \cos x) + C$$

Thus, $\int_0^1 e^x \sin x dx = \left[\frac{e^x}{2} (\sin x - \cos x) \right]_0^1 = \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2} = \frac{e(\sin 1 - \cos 1) + 1}{2} \approx 0.909.$

55. $dv = x^2 dx, v = \frac{x^3}{3}, u = \ln x, du = \frac{1}{x} dx$

$$\begin{aligned} \int x^2 \ln x dx &= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left(\frac{1}{x} \right) dx \\ &= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx \\ \text{Hence, } \int_1^2 x^2 \ln x dx &= \left[\frac{x^3}{3} \ln x - \frac{1}{9} x^3 \right]_1^2 \\ &= \frac{8}{3} \ln 2 - \frac{8}{9} + \frac{1}{9} = \frac{8}{3} \ln 2 - \frac{7}{2} \approx 1.071. \end{aligned}$$

57. $dv = x \, dx$, $v = \frac{x^2}{2}$, $u = \text{arcsec } x$, $du = \frac{1}{x\sqrt{x^2-1}} \, dx$

$$\begin{aligned} \int x \text{arcsec } x \, dx &= \frac{x^2}{2} \text{arcsec } x - \int \frac{x^2/2}{x\sqrt{x^2-1}} \, dx \\ &= \frac{x^2}{2} \text{arcsec } x - \frac{1}{4} \int \frac{2x}{\sqrt{x^2-1}} \, dx \\ &= \frac{x^2}{2} \text{arcsec } x - \frac{1}{2} \sqrt{x^2-1} + C \end{aligned}$$

Hence,

$$\begin{aligned} \int_2^4 x \text{arcsec } x \, dx &= \left[\frac{x^2}{2} \text{arcsec } x - \frac{1}{2} \sqrt{x^2-1} \right]_2^4 \\ &= \left(8 \text{arcsec } 4 - \frac{\sqrt{15}}{2} \right) - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \\ &= 8 \text{arcsec } 4 - \frac{\sqrt{15}}{2} + \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \end{aligned}$$

≈ 7.380 .

59. $\int x^2 e^{2x} \, dx = x^2 \left(\frac{1}{2} e^{2x} \right) - (2x) \left(\frac{1}{4} e^{2x} \right) + 2 \left(\frac{1}{8} e^{2x} \right) + C$

$$\begin{aligned} &= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C \\ &= \frac{1}{4} e^{2x} (2x^2 - 2x + 1) + C \end{aligned}$$

Alternate signs	u and its derivatives	v' and its antiderivatives
+	x^2	e^{2x}
-	$2x$	$\frac{1}{2} e^{2x}$
+	2	$\frac{1}{4} e^{2x}$
-	0	$\frac{1}{8} e^{2x}$

61. $\int x^3 \sin x \, dx = x^3(-\cos x) - 3x^2(-\sin x) + 6x \cos x - 6 \sin x + C$

$$\begin{aligned} &= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C \\ &= (3x^2 - 6) \sin x - (x^3 - 6x) \cos x + C \end{aligned}$$

Alternate signs	u and its derivatives	v' and its antiderivatives
+	x^3	$\sin x$
-	$3x^2$	$-\cos x$
+	$6x$	$-\sin x$
-	6	$\cos x$
+	0	$\sin x$

63. $\int x \sec^2 x \, dx = x \tan x + \ln|\cos x| + C$

Alternate signs	u and its derivatives	v' and its antiderivatives
+	x	$\sec^2 x$
-	1	$\tan x$
+	0	$-\ln \cos x $

65. Integration by parts is based on the product rule.

67. No. Substitution.

71. Yes. Let $u = x$ and $du = \frac{1}{\sqrt{x+1}} \, dx$.

(Substitution also works. Let $u = \sqrt{x+1}$)

73. $\int t^3 e^{-4t} \, dt = -\frac{e^{-4t}}{128} (32t^3 + 24t^2 + 12t + 3) + C$

75. $\int_0^{\pi/2} e^{-2x} \sin 3x \, dx = \left[\frac{e^{-2x}(-2 \sin 3x - 3 \cos 3x)}{13} \right]_0^{\pi/2} = \frac{1}{13}(2e^{-\pi} + 3) \approx 0.2374$

$$170.1 = \frac{1}{2} - \frac{1}{2} \ln \frac{8}{5} = \frac{1}{2} + \frac{2}{5} \cdot \frac{1}{2} \ln \frac{8}{5} =$$

77. (a) $dv = \sqrt{2x-3} dx \Rightarrow v = \int (2x-3)^{1/2} dx = \frac{1}{3}(2x-3)^{3/2}$

$$u = 2x \quad \Rightarrow \quad du = 2 dx$$

$$\begin{aligned}\int 2x\sqrt{2x-3} dx &= \frac{2}{3}x(2x-3)^{3/2} - \frac{2}{3} \int (2x-3)^{3/2} dx \\ &= \frac{2}{3}x(2x-3)^{3/2} - \frac{2}{15}(2x-3)^{5/2} + C \\ &= \frac{2}{15}(2x-3)^{3/2}(3x+3) + C = \frac{2}{5}(2x-3)^{3/2}(x+1) + C\end{aligned}$$

(b) $u = 2x-3 \Rightarrow x = \frac{u+3}{2}$ and $dx = \frac{1}{2}du$

$$\begin{aligned}\int 2x\sqrt{2x-3} dx &= \int 2\left(\frac{u+3}{2}\right) u^{1/2} \left(\frac{1}{2}\right) du = \frac{1}{2} \int (u^{3/2} + 3u^{1/2}) du = \frac{1}{2} \left[\frac{2}{5}u^{5/2} + 2u^{3/2} \right] + C \\ &= \frac{1}{5}u^{3/2}(u+5) + C = \frac{1}{5}(2x-3)^{3/2}[(2x-3)+5] + C = \frac{2}{5}(2x-3)^{3/2}(x+1) + C\end{aligned}$$

79. (a) $dv = \frac{x}{\sqrt{4+x^2}} dx \Rightarrow v = \int (4+x^2)^{-1/2} x dx = \sqrt{4+x^2}$

$$u = x^2 \Rightarrow du = 2x dx$$

$$\begin{aligned}\int \frac{x^3}{\sqrt{4+x^2}} dx &= x^2 \sqrt{4+x^2} - 2 \int x \sqrt{4+x^2} dx \\ &= x^2 \sqrt{4+x^2} - \frac{2}{3}(4+x^2)^{3/2} + C = \frac{1}{3}\sqrt{4+x^2}(x^2-8) + C\end{aligned}$$

(b) $u = 4+x^2 \Rightarrow x^2 = u-4$ and $2x dx = du \Rightarrow x dx = \frac{1}{2} du$

$$\begin{aligned}\int \frac{x^3}{\sqrt{4+x^2}} dx &= \int \frac{x^2}{\sqrt{4+x^2}} x dx = \int \frac{u-4}{\sqrt{u}} \frac{1}{2} du \\ &= \frac{1}{2} \int (u^{1/2} - 4u^{-1/2}) du = \frac{1}{2} \left(\frac{2}{3}u^{3/2} - 8u^{1/2} \right) + C \\ &= \frac{1}{3}u^{1/2}(u-12) + C = \frac{1}{3}\sqrt{4+x^2}[(4+x^2)-12] + C = \frac{1}{3}\sqrt{4+x^2}(x^2-8) + C\end{aligned}$$

81. $n = 0$: $\int \ln x dx = x(\ln x - 1) + C$

$$n = 1: \int x \ln x dx = \frac{x^2}{4}(2 \ln x - 1) + C$$

$$n = 2: \int x^2 \ln x dx = \frac{x^3}{9}(3 \ln x - 1) + C$$

$$n = 3: \int x^3 \ln x dx = \frac{x^4}{16}(4 \ln x - 1) + C$$

$$n = 4: \int x^4 \ln x dx = \frac{x^5}{25}(5 \ln x - 1) + C$$

In general, $\int x^n \ln x dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + C$. (See Exercise 85.)

83. $dv = \sin x dx \Rightarrow v = -\cos x$

$$u = x^n \Rightarrow du = nx^{n-1} dx$$

$$\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

85. $dv = x^n dx \Rightarrow v = \frac{x^{n+1}}{n+1}$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \ln x - \int \frac{x^n}{n+1} dx$$

$$= \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$$

$$= \frac{x^{n+1}}{(n+1)^2} [(n+1) \ln x - 1] + C$$

87. Use integration by parts twice.

(1) $dv = e^{ax} dx \Rightarrow v = \frac{1}{a} e^{ax}$

$$u = \sin bx \Rightarrow du = b \cos bx dx$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax} \sin bx}{a} - \frac{b}{a} \int e^{ax} \cos bx dx$$

$$= \frac{e^{ax} \sin bx}{a} - \frac{b}{a} \left[\frac{e^{ax} \cos bx}{a} + \frac{b}{a} \int e^{ax} \sin bx dx \right] = \frac{e^{ax} \sin bx}{a} - \frac{b^2}{a^2} \int e^{ax} \sin bx dx$$

Therefore, $\left(1 + \frac{b^2}{a^2}\right) \int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C.$$

(2) $dv = e^{ax} dx \Rightarrow v = \frac{1}{a} e^{ax}$

$$u = \cos bx \Rightarrow du = -b \sin bx dx$$

81. $\int x^3 \ln x dx$

89. $n = 3$ (Use formula in Exercise 85.)

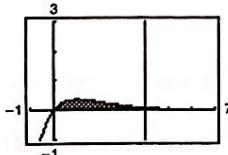
$$\int x^3 \ln x dx = \frac{x^4}{16} [4 \ln x - 1] + C$$

93. $dv = e^{-x} dx \Rightarrow v = -e^{-x}$

$$u = x \Rightarrow du = dx$$

$$A = \int_0^4 xe^{-x} dx = \left[-xe^{-x} \right]_0^4 + \int_0^4 e^{-x} dx = \frac{-4}{e^4} - \left[e^{-x} \right]_0^4$$

$$= 1 - \frac{5}{e^4} \approx 0.908$$



91. $a = 2, b = 3$ (Use formula in Exercise 88.)

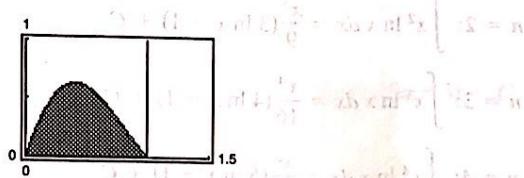
$$\int e^{2x} \cos 3x dx = \frac{e^{2x}(2 \cos 3x + 3 \sin 3x)}{13} + C$$

95. $A = \int_0^1 e^{-x} \sin(\pi x) dx$

$$= \left[\frac{e^{-x}(-\sin \pi x - \pi \cos \pi x)}{1 + \pi^2} \right]_0^1$$

$$= \frac{1}{1 + \pi^2} \left(\frac{\pi}{e} + \pi \right) = \frac{\pi}{1 + \pi^2} \left(\frac{1}{e} + 1 \right)$$

≈ 0.395 (See Exercise 87.)



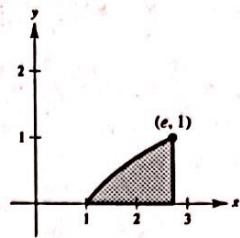
97. (a) $A = \int_1^e \ln x \, dx = \left[-x + x \ln x \right]_1^e = 1$ (See Exercise 4.)

(b) $R(x) = \ln x; r(x) = 0$

$$V = \pi \int_1^e (\ln x)^2 \, dx$$

$$= \pi \left[x(\ln x)^2 - 2x \ln x + 2x \right]_1^e \quad (\text{Use integration by parts twice, see Exercise 7.})$$

$$= \pi(e - 2) \approx 2.257$$



(c) $p(x) = x, h(x) = \ln x$

$$V = 2\pi \int_1^e x \ln x \, dx = 2\pi \left[\frac{x^2}{4} (-1 + 2 \ln x) \right]_1^e$$

$$= \frac{(e^2 + 1)\pi}{2} \approx 13.177 \quad (\text{See Exercise 85.})$$

(d) $\bar{x} = \frac{\int_1^e x \ln x \, dx}{1} = \frac{e^2 + 1}{4} \approx 2.097$

$$\bar{y} = \frac{\frac{1}{2} \int_1^e (\ln x)^2 \, dx}{1} = \frac{e - 2}{2} \approx 0.359$$

$$(\bar{x}, \bar{y}) = \left(\frac{e^2 + 1}{4}, \frac{e - 2}{2} \right) \approx (2.097, 0.359)$$

99. Average value $= \frac{1}{\pi} \int_0^\pi e^{-4t} (\cos 2t + 5 \sin 2t) \, dt$

$$= \frac{1}{\pi} \left[e^{-4t} \left(\frac{-4 \cos 2t + 2 \sin 2t}{20} \right) + 5e^{-4t} \left(\frac{-4 \sin 2t - 2 \cos 2t}{20} \right) \right]_0^\pi \quad (\text{From Exercises 87 and 88})$$

$$= \frac{7}{10\pi} (1 - e^{-4\pi}) \approx 0.223$$

101. $c(t) = 100,000 + 4000t, r = 5\%, t_1 = 10$

$$P = \int_0^{10} (100,000 + 4000t) e^{-0.05t} \, dt = 4000 \int_0^{10} (25 + t) e^{-0.05t} \, dt$$

Let $u = 25 + t, dv = e^{-0.05t} \, dt, du = dt, v = -\frac{100}{5} e^{-0.05t}$

$$P = 4000 \left\{ \left[(25 + t) \left(-\frac{100}{5} e^{-0.05t} \right) \right]_0^{10} + \frac{100}{5} \int_0^{10} e^{-0.05t} \, dt \right\}$$

$$= 4000 \left\{ \left[(25 + t) \left(-\frac{100}{5} e^{-0.05t} \right) \right]_0^{10} - \left[\frac{10,000}{25} e^{-0.05t} \right]_0^{10} \right\} \approx \$931,265$$

103. $\int_{-\pi}^{\pi} x \sin nx \, dx = \left[-\frac{x}{n} \cos nx + \frac{1}{n^2} \sin nx \right]_{-\pi}^{\pi}$

$$= -\frac{\pi}{n} \cos \pi n - \frac{\pi}{n} \cos(-\pi n)$$

$$= -\frac{2\pi}{n} \cos \pi n$$

$$= \begin{cases} -(2\pi/n), & \text{if } n \text{ is even} \\ (2\pi/n), & \text{if } n \text{ is odd} \end{cases}$$

105. Let $u = x$, $dv = \sin\left(\frac{n\pi}{2}x\right)dx$, $du = dx$, $v = -\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}x\right)$.

$$\begin{aligned} I_1 &= \int_0^1 x \sin\left(\frac{n\pi}{2}x\right) dx = \left[\frac{-2x}{n\pi} \cos\left(\frac{n\pi}{2}x\right) \right]_0^1 + \frac{2}{n\pi} \int_0^1 \cos\left(\frac{n\pi}{2}x\right) dx \\ &= -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \left[\left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}x\right) \right]_0^1 \\ &= -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right) \end{aligned}$$

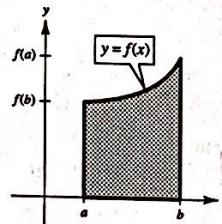
Let $u = (-x + 2)$, $dv = \sin\left(\frac{n\pi}{2}x\right)dx$, $du = -dx$, $v = -\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}x\right)$.

$$\begin{aligned} I_2 &= \int_1^2 (-x + 2) \sin\left(\frac{n\pi}{2}x\right) dx = \left[\frac{-2(-x + 2)}{n\pi} \cos\left(\frac{n\pi}{2}x\right) \right]_1^2 - \frac{2}{n\pi} \int_1^2 \cos\left(\frac{n\pi}{2}x\right) dx \\ &= \frac{2}{n\pi} \cos\left(\frac{n\pi}{2}\right) - \left[\left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}x\right) \right]_1^2 \\ &= \frac{2}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right) \end{aligned}$$

$$h(I_1 + I_2) = b_n = h \left[\left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right) + \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi}{2}\right) \right] = \frac{8h}{(n\pi)^2} \sin\left(\frac{n\pi}{2}\right)$$

107. Shell Method:

$$\begin{aligned} V &= 2\pi \int_a^b x f'(x) dx \\ dv &= x dx \Rightarrow v = \frac{x^2}{2} \\ u &= f(x) \Rightarrow du = f'(x) dx \\ V &= 2\pi \left[\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x) dx \right]_a^b \\ &= \pi \left[(b^2 f(b) - a^2 f(a)) - \int_a^b x^2 f'(x) dx \right] \end{aligned}$$



Disk Method:

$$\begin{aligned} V &= \pi \int_0^{f(a)} (b^2 - a^2) dy + \pi \int_{f(a)}^{f(b)} [b^2 - [f^{-1}(y)]^2] dy \\ &= \pi(b^2 - a^2)f(a) + \pi b^2(f(b) - f(a)) - \pi \int_{f(a)}^{f(b)} [f^{-1}(y)]^2 dy \\ &= \pi \left[(b^2 f(b) - a^2 f(a)) - \int_{f(a)}^{f(b)} [f^{-1}(y)]^2 dy \right] \end{aligned}$$

Since $x = f^{-1}(y)$, we have $f(x) = y$ and $f'(x)dx = dy$. When $y = f(a)$, $x = a$. When $y = f(b)$, $x = b$. Thus,

$$\int_{f(a)}^{f(b)} [f^{-1}(y)]^2 dy = \int_a^b x^2 f'(x) dx$$

and the volumes are the same.

109. $f'(x) = xe^{-x}$

(a) $f(x) = \int xe^{-x} dx = -xe^{-x} - e^{-x} + C$

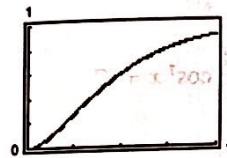
(Parts: $u = x$, $dv = e^{-x} dx$)

$f(0) = 0 = -1 + C \Rightarrow C = 1$

$f(x) = -xe^{-x} - e^{-x} + 1$

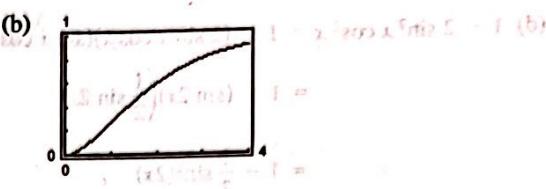
(c) You obtain the points

n	x_n	y_n
0	0	0
1	0.05	0
2	0.10	2.378×10^{-3}
3	0.15	0.0069
4	0.20	0.0134
\vdots	\vdots	\vdots
80	4.0	0.9064



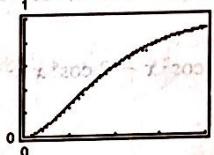
(e) $f(4) = 0.9084$

The approximations are tangent line approximations. The results in (c) are better because Δx is smaller.



(d) You obtain the points

n	x_n	y_n
0	0	0
1	0.1	0
2	0.2	0.0090484
3	0.3	0.025423
4	0.4	0.047648
\vdots	\vdots	\vdots
40	4.0	0.9039



Section 7.3 Trigonometric Integrals

1. $f(x) = \sin^4 x + \cos^4 x$

(a) $\sin^4 x + \cos^4 x = \left(\frac{1 - \cos 2x}{2}\right)^2 + \left(\frac{1 + \cos 2x}{2}\right)^2$

$$= \frac{1}{4}[1 - 2\cos 2x + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x]$$

$$= \frac{1}{4}[2 + 2\frac{1 + \cos 4x}{2}]$$

$$= \frac{1}{4}[3 + \cos 4x]$$

(b) $\sin^4 x + \cos^4 x = (\sin^2 x)^2 + \cos^4 x$

$$= (1 - \cos^2 x)^2 + \cos^4 x$$

$$= 1 - 2\cos^2 x + 2\cos^4 x$$

(c) $\sin^4 x + \cos^4 x = \sin^4 x + 2\sin^2 x \cos^2 x + \cos^4 x - 2\sin^2 x \cos^2 x$

$$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$$

$$= 1 - 2\sin^2 x \cos^2 x$$

—CONTINUED—