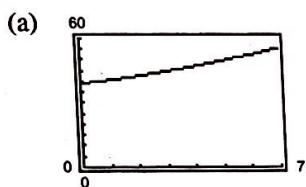


74. (a) $F_{\text{inside}} = 48 \int_{-1}^{0.8} (0.8 - y)(2)\sqrt{1 - y^2} dy$
 $= 96 \left[0.8 \int_{-1}^{0.8} \sqrt{1 - y^2} dy - \int_{-1}^{0.8} y \sqrt{1 - y^2} dy \right]$
 $= 96 \left[\frac{0.8}{2} (\arcsin y + y\sqrt{1 - y^2}) + \frac{1}{3}(1 - y^2)^{3/2} \right]_{-1}^{0.8} \approx 96(1.263) \approx 121.3 \text{ lbs}$

(b) $F_{\text{outside}} = 64 \int_{-1}^{0.4} (0.4 - y)(2)\sqrt{1 - y^2} dy$
 $= 128 \left[0.4 \int_{-1}^{0.4} \sqrt{1 - y^2} dy - \int_{-1}^{0.4} y \sqrt{1 - y^2} dy \right]$
 $= 128 \left[\frac{0.4}{2} (\arcsin y + y\sqrt{1 - y^2}) + \frac{1}{3}(1 - y^2)^{3/2} \right]_{-1}^{0.4} \approx 92.98$

76. $S = \sqrt{1520.4 + 111.2t + 15.8t^2}$



(b) $S'(t) = \frac{1}{2}(1520.4 + 111.2t + 15.8t^2)^{-1/2}(111.2 + 31.6t)$

$S'(5) \approx 2.71$

(c) Average value = $\frac{1}{2} \int_{10}^{12} S(t) dt \approx 68.24$

80. True

$$\int_{-1}^1 x^2 \sqrt{1 - x^2} dx = 2 \int_0^1 x^2 \sqrt{1 - x^2} dx = 2 \int_0^{\pi/2} (\sin^2 \theta)(\cos \theta)(\cos \theta d\theta) = 2 \int_0^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta$$

Section 7.5 Partial Fractions

2. $\frac{4x^2 + 3}{(x - 5)^3} = \frac{A}{x - 5} + \frac{B}{(x - 5)^2} + \frac{C}{(x - 5)^3}$

4. $\frac{x - 2}{x^2 + 4x + 3} = \frac{x - 2}{(x + 1)(x + 3)} = \frac{A}{x + 1} + \frac{B}{x + 3}$

6. $\frac{2x - 1}{x(x^2 + 1)^2} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$

8. $\frac{1}{4x^2 - 9} = \frac{1}{(2x - 3)(2x + 3)} = \frac{A}{2x - 3} + \frac{B}{2x + 3}$
 $1 = A(2x + 3) + B(2x - 3)$

When $x = \frac{3}{2}$, $1 = 6A$, $A = \frac{1}{6}$.

When $x = -\frac{3}{2}$, $1 = -6B$, $B = -\frac{1}{6}$.

$$\begin{aligned} \int \frac{1}{4x^2 - 9} dx &= \frac{1}{6} \left[\int \frac{1}{2x - 3} dx - \int \frac{1}{2x + 3} dx \right] \\ &= \frac{1}{12} [\ln|2x - 3| - \ln|2x + 3|] + C \\ &= \frac{1}{12} \ln \left| \frac{2x - 3}{2x + 3} \right| + C \end{aligned}$$

10. $\int \frac{x + 1}{x^2 + 4x + 3} dx = \int \frac{(x + 1)}{(x + 1)(x + 3)} dx$
 $= \int \frac{1}{x + 3} dx = \ln|x + 3| + C$

12. $\frac{5x^2 - 12x - 12}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$

$$5x^2 - 12x - 12 = A(x^2 - 4) + Bx(x+2) + Cx(x-2)$$

When $x = 0, -12 = -4A \Rightarrow A = 3$. When $x = 2, -16 = 8B \Rightarrow B = -2$. When $x = -2, 32 = 8C \Rightarrow C = 4$.

$$\int \frac{5x^2 - 12x - 12}{x^3 - 4x} dx = \int \frac{3}{x} dx + \int \frac{-2}{x-2} dx + \int \frac{4}{x+2} dx = 3 \ln|x| - 2 \ln|x-2| + 4 \ln|x+2| + C$$

14. $\frac{x^3 - x + 3}{x^2 + x - 2} = x - 1 + \frac{2x + 1}{(x+2)(x-1)} = x - 1 + \frac{A}{x+2} + \frac{B}{x-1}$

$$2x + 1 = A(x-1) + B(x+2)$$

When $x = -2, -3 = -3A, A = 1$. When $x = 1, 3 = 3B, B = 1$.

$$\begin{aligned} \int \frac{x^3 - x + 3}{x^2 + x - 2} dx &= \int \left[x - 1 + \frac{1}{x+2} + \frac{1}{x-1} \right] dx \\ &= \frac{x^2}{2} - x + \ln|x+2| + \ln|x-1| + C = \frac{x^2}{2} - x + \ln|x^2 + x - 2| + C \end{aligned}$$

16. $\frac{x+2}{x(x-4)} = \frac{A}{x-4} + \frac{B}{x}$

$$x+2 = Ax + B(x-4)$$

When $x = 4, 6 = 4A, A = \frac{3}{2}$.

When $x = 0, 2 = -4B, B = -\frac{1}{2}$.

$$\begin{aligned} \int \frac{x+2}{x^2 - 4x} dx &= \int \left[\frac{3/2}{x-4} - \frac{1/2}{x} \right] dx \\ &= \frac{3}{2} \ln|x-4| - \frac{1}{2} \ln|x| + C \end{aligned}$$

18. $\frac{2x-3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$$2x-3 = A(x-1) + B$$

When $x = 1, B = -1$. When $x = 0, A = 2$.

$$\begin{aligned} \int \frac{2x-3}{(x-1)^2} dx &= \int \left[\frac{2}{x-1} - \frac{1}{(x-1)^2} \right] dx \\ &= 2 \ln|x-1| + \frac{1}{x-1} + C \end{aligned}$$

20. $\frac{4x^2}{x^3 + x^2 - x - 1} = \frac{4x^2}{x^2(x+1) - (x+1)} = \frac{4x^2}{(x^2-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$$4x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

When $x = -1, 4 = -2C \Rightarrow C = -2$. When $x = 1, 4 = 4A \Rightarrow A = 1$. When $x = 0, 0 = 1 - B + 2 \Rightarrow B = 3$.

$$\begin{aligned} \int \frac{4x^2}{x^3 + x^2 - x - 1} dx &= \int \frac{1}{x-1} dx + \int \frac{3}{x+1} dx - \int \frac{2}{(x+1)^2} dx \\ &= \ln|x-1| + 3 \ln|x+1| + \frac{2}{(x+1)} + C \end{aligned}$$

22. $\frac{6x}{x^3 - 8} = \frac{6x}{(x-2)(x^2 + 2x + 4)} = \frac{A}{x-2} + \frac{Bx + C}{x^2 + 2x + 4}$

$$6x = A(x^2 + 2x + 4) + (Bx + C)(x-2)$$

When $x = 2, 12 = 12A \Rightarrow A = 1$. When $x = 0, 0 = 4 - 2C \Rightarrow C = 2$. When $x = 1, 6 = 7 + (B+2)(-1) \Rightarrow B = -1$.

$$\begin{aligned} \int \frac{6x}{x^3 - 8} dx &= \int \frac{1}{x-2} dx + \int \frac{-x+2}{x^2 + 2x + 4} dx = \int \frac{1}{x-2} dx + \int \frac{-x-1}{x^2 + 2x + 4} dx + \int \frac{3}{(x^2 + 2x + 1) + 3} dx \\ &= \ln|x-2| - \frac{1}{2} \ln|x^2 + 2x + 4| + \frac{3}{\sqrt{3}} \arctan\left(\frac{x+1}{\sqrt{3}}\right) + C \end{aligned}$$

$$\begin{aligned} &= \ln|x-2| - \frac{1}{2} \ln|x^2 + 2x + 4| + \sqrt{3} \arctan\left(\frac{\sqrt{3}(x+1)}{3}\right) + C \end{aligned}$$

24. $\frac{x^2 - x + 9}{(x^2 + 9)^2} = \frac{Ax + B}{x^2 + 9} + \frac{Cx + D}{(x^2 + 9)^2}$

$$\begin{aligned} x^2 - x + 9 &= (Ax + B)(x^2 + 9) + Cx + D \\ &= Ax^3 + Bx^2 + (9A + C)x + (9B + D) \end{aligned}$$

By equating coefficients of like terms, we have $A = 0$, $B = 1$, $D = 0$, and $C = -1$.

$$\begin{aligned} \int \frac{x^2 - x + 9}{(x^2 + 9)^2} dx &= \int \frac{1}{x^2 + 9} dx - \int \frac{x}{(x^2 + 9)^2} dx \\ &= \frac{1}{3} \arctan\left(\frac{x}{3}\right) + \frac{1}{2(x^2 + 9)} + C \end{aligned}$$

26. $\frac{x^2 - 4x + 7}{(x+1)(x^2 - 2x + 3)} = \frac{A}{x+1} + \frac{Bx + C}{x^2 - 2x + 3}$

$$x^2 - 4x + 7 = A(x^2 - 2x + 3) + (Bx + C)(x + 1)$$

When $x = -1$, $12 = 6A$. When $x = 0$, $7 = 3A + C$. When $x = 1$, $4 = 2A + 2B + 2C$. Solving these equations we have $A = 2$, $B = -1$, $C = 1$.

$$\begin{aligned} \int \frac{x^2 - 4x + 7}{x^3 - x^2 + x + 3} dx &= 2 \int \frac{1}{x+1} dx + \int \frac{-x+1}{x^2 - 2x + 3} dx \\ &= 2 \ln|x+1| - \frac{1}{2} \ln|x^2 - 2x + 3| + C \end{aligned}$$

28. $\frac{x^2 + x + 3}{(x^2 + 3)^2} = \frac{Ax + B}{x^2 + 3} + \frac{Cx + D}{(x^2 + 3)^2}$

$$\begin{aligned} x^2 + x + 3 &= (Ax + B)(x^2 + 3) + Cx + D \\ &= Ax^3 + Bx^2 + (3A + C)x + (3B + D) \end{aligned}$$

By equating coefficients of like terms, we have $A = 0$, $B = 1$, $3A + C = 1$, $3B + D = 3$. Solving these equations we have $A = 0$, $B = 1$, $C = 1$, $D = 0$.

$$\begin{aligned} \int \frac{x^2 + x + 3}{x^4 + 6x^2 + 9} dx &= \int \left[\frac{1}{x^2 + 3} + \frac{x}{(x^2 + 3)^2} \right] dx \\ &= \frac{1}{\sqrt{3}} \arctan\frac{x}{\sqrt{3}} - \frac{1}{2(x^2 + 3)} + C \end{aligned}$$

30. $\frac{x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

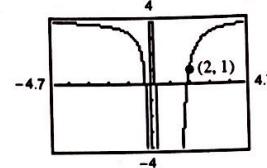
$$x-1 = Ax(x+1) + B(x+1) + Cx^2$$

When $x = 0$, $B = -1$. When $x = -1$, $C = -2$. When $x = 1$, $0 = 2A + 2B + C$. Solving these equations we have $A = 2$, $B = -1$, $C = -2$.

$$\begin{aligned} \int_1^5 \frac{x-1}{x^2(x+1)} dx &= 2 \int_1^5 \frac{1}{x} dx - \int_1^5 \frac{1}{x^2} dx - 2 \int_1^5 \frac{1}{x+1} dx \\ &= \left[2 \ln|x| + \frac{1}{x} - 2 \ln|x+1| \right]_1^5 \\ &= \left[2 \ln\left|\frac{x}{x+1}\right| + \frac{1}{x} \right]_1^5 \\ &= 2 \ln\frac{5}{3} - \frac{4}{5} \end{aligned}$$

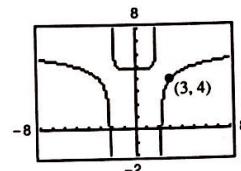
32. $\int_0^1 \frac{x^2 - x}{x^2 + x + 1} dx = \int_0^1 dx - \int_0^1 \frac{2x + 1}{x^2 + x + 1} dx = \left[x - \ln|x^2 + x + 1| \right]_0^1 = 1 - \ln 3$

34. $\int \frac{6x^2 + 1}{x^2(x-1)^3} dx = 3 \ln \left| \frac{x-1}{x} \right| + \frac{1}{x} + \frac{2}{x-1} - \frac{7}{2(x-1)^2} + C$
 $(2, 1): 3 \ln \left| \frac{1}{2} \right| + \frac{1}{2} + \frac{2}{1} - \frac{7}{2} + C = 1 \Rightarrow C = 2 - 3 \ln \frac{1}{2}$



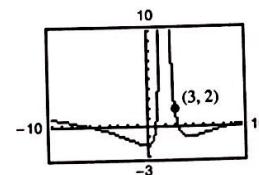
36. $\int \frac{x^3}{(x^2 - 4)^2} dx = \frac{1}{2} \ln|x^2 - 4| - \frac{2}{x^2 - 4} + C$

$(3, 4): \frac{1}{2} \ln 5 - \frac{2}{5} + C = 4 \Rightarrow C = \frac{22}{5} - \frac{1}{2} \ln 5$



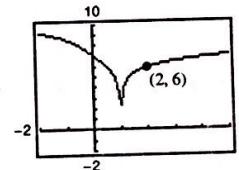
38. $\int \frac{x(2x-9)}{x^3 - 6x^2 + 12x - 8} dx = 2 \ln|x-2| + \frac{1}{x-2} + \frac{5}{(x-2)^2} + C$

$(3, 2): 0 + 1 + 5 + C = 2 \Rightarrow C = -4$



40. $\int \frac{x^2 - x + 2}{x^3 - x^2 + x - 1} dx = -\arctan x + \ln|x-1| + C$

$(2, 6): -\arctan 2 + 0 + C = 6 \Rightarrow C = 6 + \arctan 2$



42. Let $u = \cos x$, $du = -\sin x dx$.

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$$1 = A(u+1) + Bu$$

When $u = 0, A = 1$. When $u = -1, B = -1$, $u = \cos x$.
 $du = -\sin x dx$.

$$\begin{aligned} \int \frac{\sin x}{\cos x + \cos^2 x} dx &= - \int \frac{1}{u(u+1)} du \\ &= \int \frac{1}{u+1} du - \int \frac{1}{u} du \\ &= \ln|u+1| - \ln|u| + C \\ &= \ln \left| \frac{u+1}{u} \right| + C \\ &= \ln \left| \frac{\cos x + 1}{\cos x} \right| + C \\ &= \ln|1 + \sec x| + C \end{aligned}$$

44. $\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}, u = \tan x, du = \sec^2 x dx$
 $1 = A(u+1) + Bu$

When $u = 0, A = 1$.

When $u = -1, 1 = -B \Rightarrow B = -1$.

$$\begin{aligned} \int \frac{\sec^2 x dx}{\tan x(\tan x + 1)} &= \int \frac{1}{u(u+1)} du \\ &= \int \left(\frac{1}{u} - \frac{1}{u+1} \right) du \\ &= \ln|u| - \ln|u+1| + C \\ &= \ln \left| \frac{u}{u+1} \right| + C \\ &= \ln \left| \frac{\tan x}{\tan x + 1} \right| + C \end{aligned}$$