

3. Infinite discontinuity at $x = 1$.

$$\begin{aligned} \int_0^2 \frac{1}{(x-1)^2} dx &= \int_0^1 \frac{1}{(x-1)^2} dx + \int_1^2 \frac{1}{(x-1)^2} dx \\ &= \lim_{b \rightarrow 1^-} \int_0^b \frac{1}{(x-1)^2} dx + \lim_{c \rightarrow 1^+} \int_c^2 \frac{1}{(x-1)^2} dx \\ &= \lim_{b \rightarrow 1^-} \left[-\frac{1}{x-1} \right]_0^b + \lim_{c \rightarrow 1^+} \left[-\frac{1}{x-1} \right]_c^2 = (\infty - 1) + (-1 + \infty) \end{aligned}$$

Diverges

5. Infinite limit of integration.

$$\begin{aligned} \int_0^\infty e^{-x} dx &= \lim_{b \rightarrow \infty} \int_0^b e^{-x} dx \\ &= \lim_{b \rightarrow \infty} \left[-e^{-x} \right]_0^b = 0 + 1 = 1 \end{aligned}$$

Converges

$$\begin{aligned} 9. \int_1^\infty \frac{1}{x^2} dx &= \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} dx \\ &= \lim_{b \rightarrow \infty} \left[-\frac{1}{x} \right]_1^b = 1 \end{aligned}$$

$$7. \int_{-1}^1 \frac{1}{x^2} dx \neq -2$$

because the integrand is not defined at $x = 0$.

Diverges

$$\begin{aligned} 11. \int_1^\infty \frac{3}{\sqrt[3]{x}} dx &= \lim_{b \rightarrow \infty} \int_1^b 3x^{-1/3} dx \\ &= \lim_{b \rightarrow \infty} \left[\frac{9}{2} x^{2/3} \right]_1^b = \infty \end{aligned}$$

Diverges

$$13. \int_{-\infty}^0 xe^{-2x} dx = \lim_{b \rightarrow -\infty} \int_b^0 xe^{-2x} dx = \lim_{b \rightarrow -\infty} \frac{1}{4} \left[(-2x-1)e^{-2x} \right]_b^0 = \lim_{b \rightarrow -\infty} \frac{1}{4} [-1 + (2b+1)e^{-2b}] = -\infty \quad (\text{Integration by parts})$$

Diverges

$$15. \int_0^\infty x^2 e^{-x} dx = \lim_{b \rightarrow \infty} \int_0^b x^2 e^{-x} dx = \lim_{b \rightarrow \infty} \left[-e^{-x}(x^2 + 2x + 2) \right]_0^b = \lim_{b \rightarrow \infty} \left(-\frac{b^2 + 2b + 2}{e^b} + 2 \right) = 2$$

Since $\lim_{b \rightarrow \infty} \left(-\frac{b^2 + 2b + 2}{e^b} \right) = 0$ by L'Hôpital's Rule.

$$\begin{aligned} 17. \int_0^\infty e^{-x} \cos x dx &= \lim_{b \rightarrow \infty} \frac{1}{2} \left[e^{-x}(-\cos x + \sin x) \right]_0^b \\ &= \frac{1}{2} [0 - (-1)] = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 19. \int_4^\infty \frac{1}{x(\ln x)^3} dx &= \lim_{b \rightarrow \infty} \int_4^b (\ln x)^{-3} \frac{1}{x} dx \\ &= \lim_{b \rightarrow \infty} \left[-\frac{1}{2} (\ln x)^{-2} \right]_4^b \\ &= -\frac{1}{2} (\ln b)^{-2} + \frac{1}{2} (\ln 4)^{-2} \\ &= \frac{1}{2} \frac{1}{(2 \ln 2)^2} = \frac{1}{8(\ln 2)^2} \end{aligned}$$

$$\begin{aligned} 21. \int_{-\infty}^\infty \frac{2}{4+x^2} dx &= \int_{-\infty}^0 \frac{2}{4+x^2} dx + \int_0^\infty \frac{2}{4+x^2} dx \\ &= \lim_{b \rightarrow -\infty} \int_b^0 \frac{2}{4+x^2} dx + \lim_{c \rightarrow \infty} \int_0^c \frac{2}{4+x^2} dx \\ &= \lim_{b \rightarrow -\infty} \left[\arctan\left(\frac{x}{2}\right) \right]_b^0 + \lim_{c \rightarrow \infty} \left[\arctan\left(\frac{x}{2}\right) \right]_0^c \\ &= \left(0 - \left(-\frac{\pi}{2} \right) \right) + \left(\frac{\pi}{2} - 0 \right) = \pi \end{aligned}$$

$$23. \int_0^\infty \frac{1}{e^x + e^{-x}} dx = \lim_{b \rightarrow \infty} \int_0^b \frac{e^x}{1 + e^{2x}} dx \\ = \lim_{b \rightarrow \infty} \left[\arctan(e^x) \right]_0^b \\ = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$27. \int_0^1 \frac{1}{x^2} dx = \lim_{b \rightarrow 0^+} \left[\frac{-1}{x} \right]_0^1 = -1 + \infty$$

Diverges

$$29. \int_0^8 \frac{1}{\sqrt[3]{8-x}} dx = \lim_{b \rightarrow 8^-} \int_0^b \frac{1}{\sqrt[3]{8-x}} dx = \lim_{b \rightarrow 8^-} \left[\frac{-3}{2}(8-x)^{2/3} \right]_0^b = 6$$

$$31. \int_0^1 x \ln x dx = \lim_{b \rightarrow 0^+} \left[\frac{x^2}{2} \ln|x| - \frac{x^2}{4} \right]_b^1 = \lim_{b \rightarrow 0^+} \left[\frac{-1}{4} - \frac{b^2 \ln b}{2} + \frac{b^2}{4} \right] = \frac{-1}{4} \text{ since } \lim_{b \rightarrow 0^+} (b^2 \ln b) = 0 \text{ by L'Hôpital's Rule.}$$

$$33. \int_0^{\pi/2} \tan \theta d\theta = \lim_{b \rightarrow (\pi/2)^-} \left[\ln|\sec \theta| \right]_0^b = \infty$$

Diverges

$$35. \int_2^4 \frac{2}{x\sqrt{x^2-4}} dx = \lim_{b \rightarrow 2^+} \int_b^4 \frac{2}{x\sqrt{x^2-4}} dx \\ = \lim_{b \rightarrow 2^+} \left[\operatorname{arcsec} \left| \frac{x}{2} \right| \right]_b^4 \\ = \lim_{b \rightarrow 2^+} \left(\operatorname{arcsec} 2 - \operatorname{arcsec} \left(\frac{b}{2} \right) \right) \\ = \frac{\pi}{3} - 0 = \frac{\pi}{3}$$

$$37. \int_2^4 \frac{1}{\sqrt{x^2-4}} dx = \lim_{b \rightarrow 2^+} \left[\ln|x + \sqrt{x^2-4}| \right]_b^4 \\ = \ln(4 + 2\sqrt{3}) - \ln 2 \\ = \ln(2 + \sqrt{3}) \approx 1.317$$

$$39. \int_0^2 \frac{1}{\sqrt[3]{x-1}} dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}} dx + \int_1^2 \frac{1}{\sqrt[3]{x-1}} dx \\ = \lim_{b \rightarrow 1^-} \left[\frac{3}{2}(x-1)^{2/3} \right]_0^b + \lim_{c \rightarrow 1^+} \left[\frac{3}{2}(x-1)^{2/3} \right]_c^2 = \frac{-3}{2} + \frac{3}{2} = 0$$

$$41. \int_0^\infty \frac{4}{\sqrt{x(x+6)}} dx = \int_0^1 \frac{4}{\sqrt{x(x+6)}} dx + \int_1^\infty \frac{4}{\sqrt{x(x+6)}} dx$$

Let $u = \sqrt{x}$, $u^2 = x$, $2u du = dx$.

$$\int \frac{4}{\sqrt{x(x+6)}} dx = \int \frac{4(2u du)}{u(u^2+6)} = 8 \int \frac{du}{u^2+6} = \frac{8}{\sqrt{6}} \arctan \left(\frac{u}{\sqrt{6}} \right) + C = \frac{8}{\sqrt{6}} \arctan \left(\frac{\sqrt{x}}{\sqrt{6}} \right) + C$$

$$\text{Thus, } \int_0^\infty \frac{4}{\sqrt{x(x+6)}} dx = \lim_{b \rightarrow 0^+} \left[\frac{8}{\sqrt{6}} \arctan \left(\frac{\sqrt{x}}{\sqrt{6}} \right) \right]_b^1 + \lim_{c \rightarrow \infty} \left[\frac{8}{\sqrt{6}} \arctan \left(\frac{\sqrt{x}}{\sqrt{6}} \right) \right]_1^c \\ = \left(\frac{8}{\sqrt{6}} \arctan \left(\frac{1}{\sqrt{6}} \right) - \frac{8}{\sqrt{6}} 0 \right) + \left(\frac{8}{\sqrt{6}} \frac{\pi}{2} - \frac{8}{\sqrt{6}} \arctan \left(\frac{1}{\sqrt{6}} \right) \right) \\ = \frac{8\pi}{2\sqrt{6}} = \frac{2\pi\sqrt{6}}{3}$$

$$25. \int_0^\infty \cos \pi x dx = \lim_{b \rightarrow \infty} \left[\frac{1}{\pi} \sin \pi x \right]_0^b$$

Diverges since $\sin \pi x$ does not approach a limit as $x \rightarrow \infty$.