8.1 Sequences

* domain (n) is the set of Natural numbers

$\Rightarrow n$ = # of the term $n = 1, 2, 3, 4, \ldots$

$a_n \Rightarrow$ value of the n^{th} term (like "y")

a_n = explicit formula so very similar to $f(x) =$ ___

$\{a_n\} \rightarrow$ sequence

Ex $\{3n-1\} = \overset{n=1}{2}, \overset{n=2}{5}, \overset{n=3}{8}, \ldots, 3n-1, \ldots$

* Converge or diverge?

If you have infinite number of terms, do the values of the terms start to approach one #?

A sequence converges if $\lim_{n\to\infty} \{a_n\} = L$ $(L = \#)$

A sequence diverges if $\lim_{n\to\infty} \{a_n\} =$ DNE or $\pm\infty$

* $\{(-1)^n a_n\} \Rightarrow$ alternating between + and −

Only converges if $\lim_{n\to\infty} a_n = 0$.

* If exponential ÷ exponential, rewrite with one exponent.

Ex. $\frac{4^{n+1}}{5^{2n}} = \frac{4 \cdot 4^n}{25^n} = 4\left(\frac{4}{25}\right)^n$ ← if base < 1 then $\lim_{n\to\infty} 4\left(\frac{4}{25}\right)^n = 0$

Determine if sequence converges or diverges?

1. $\{3n^2-3\}$

$\lim_{n\to\infty} 3n^2-3 = \infty$

Diverges

2. $\left\{\frac{5n}{e^n}\right\}$

$\lim_{n\to\infty}\left\{\frac{5n}{e^n}\right\} = 0$

Converges to 0

3. $\left\{\frac{2n^2}{5n^2-3}\right\}$.

$\lim_{n\to\infty} \frac{2n^2}{5n^2-3} = \frac{2}{5}$

converges to $\frac{2}{5}$

4. $\left\{\frac{2^n}{e^{2n}}\right\}$

Rewrite $\left\{\left(\frac{2}{e^2}\right)^n\right\}$

$\lim_{n\to\infty}\left(\frac{2}{e^2}\right)^n = 0$ ← base <1

Converges

5. $\left\{\frac{4^{-n+1}}{5^{-2n}}\right\} \Rightarrow \frac{4^{-n}\cdot 4^1}{5^{-2n}} = \frac{5^{2n}\cdot 4}{4^n}$

$= 4\left(\frac{25}{4}\right)^n$

$\lim_{n\to\infty} 4\left(\frac{25}{4}\right)^n \Rightarrow \infty$

b/c $b > 1$

diverges

6. $\{3+(-1)^n\}$

n is odd $\Rightarrow 2$

n is even $\Rightarrow 4$

$\lim_{n\to\infty} 3+(-1)^n = \text{DNE}$

Diverges

7. $\left\{\frac{(-1)^n}{n^2}\right\}$

$\lim_{n\to\infty} \frac{(-1)^n}{n^2} = 0$

Converges to 0.

8. $\left\{\frac{(-1)^n 2n^2}{3n^2-8}\right\}$

DNE

Diverges

(limit alternates b/t $\pm 2/3$ so DNE)

$\lim_{n\to\infty} \left\{ \frac{\cos^2 n}{3^n} \right\} = 0$

* $\{(-1)^n a_n\}$ ← alternating

Monotonic – strictly increasing or strictly decreasing

bounded – bounded both above and below

Thm If a sequence is bounded and monotonic then it converges.

Factorials (!)

$0! = 1$

$n! = n(n-1)(n-2)\cdots 1$

$4! = 4\cdot 3\cdot 2\cdot 1$

$7! = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

$2n! = 2\cdot n\cdot (n-1)(n-2)\cdots$

$(2n)! = (2n)(2n-1)(2n-2)\cdots$

Can simplify factorials $\Rightarrow \frac{6!}{4!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{4\cdot 3\cdot 2\cdot 1} = 30$

$\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)(n)(n-1)\cdots}{n(n-1)\cdots} = (n+2)(n+1)$

HW: p. 564 #3-15m3, 17-20, 27-41odd, 47-65all, 67-75