Length of Circular Arc

In a circle of radius, r, the length s of an arc that subtends a central angle of θ radians is:

$$s = r\theta$$

*True if and only if θ is in radians! If the angle given is in degree measure, use your conversion rule from yesterday to change the angle in degree measure to radian measure by multiplying by $\frac{\pi}{180}$.

Examples:

1. Find the length of an arc of a circle with radius 10 m that subtends a central angle of 30º.

$S = r\theta \quad S = (10)\left(\frac{\pi}{6}\right) = \frac{5\pi}{3} = \boxed{5.24\text{ m}} \qquad 30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$

2. A central angle θ in a circle of radius 4 m is subtended by an arc of length 6 m. Find the measure of θ in radians and in degrees.

$S = r\theta \quad \theta = \frac{6}{4} = 1.5\text{ rad.} \qquad 1.5 \times \frac{180}{\pi} = \boxed{85.94^\circ}$

$6 = 4\theta$

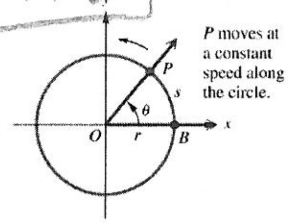
3. Memphis, TN and New Orleans, LA lie approximately on the same meridian. Memphis has latitude 35ºN and New Orleans 30ºN. Find the distance between the two cities. (Radius of earth is 3960 miles)

$\theta = 5^\circ \cdot \frac{\pi}{180} = \pi/36$

$S = 3960(\pi/36) = \boxed{345.58\text{ miles}}$

Angular and Linear Speed (Velocity)

https://www.youtube.com/watch?v=jh9gRYAuau8



Sometimes it is important to know how fast a point is moving (____________________) or how fast a central angle is changing (____________________)

Linear Speed (Velocity) when not a circular motion can be found by d = rt. Or **d = vt**

Angular Speed (Velocity): The measure of how fast an angle is changing, *angular velocity*, ω (omega)

$$\omega = \frac{\theta}{t} \qquad \textit{where } \theta \textit{ is measured of angle in radians at time, } t$$

This is often expressed in rpms (or revolutions per minute). To be angular speed, you need to convert to radians per time.

Example: A record rotates at a rate of 50 rpms. Determine the angular speed in radians per second. (One rotation is 360 degrees, or 2π radians—change to radians)

$\frac{50\text{ rev.}}{1\text{ min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{100\pi}{60}\text{ rad/sec} = \frac{5\pi}{3}\text{ rad/sec}$

$= 5.236\text{ rad/s}$

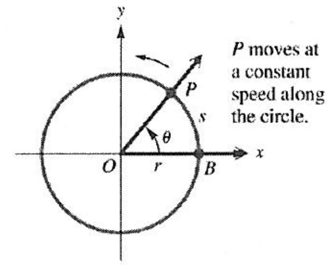
$d = rt \quad r = \frac{d}{t} = \frac{s}{t} = \frac{r\theta}{t}$

Linear Speed (Velocity) around a circle: If P is a point on a circle of radius r, and P moves a distance s *(arclength)* on the circumference of the circle in an amount of time t, then you can find the linear velocity (v) if you replace the distance with the arclength traveled (s).

$v = \frac{s}{t}$ but since s = rΘ, we can know that

$$v = \frac{r\theta}{t} \quad or \quad v = r\omega \quad or \quad v = r\frac{\theta}{t}$$

$\frac{\theta}{t}$ → angular velocity ω



If a point is moving with uniform circular motion on a circle of radius r, then the linear velocity v and angular velocity ω of the point are related by the formula

$$v = r\omega$$

Example: A tire with radius of 9 inches is spinning at 80 revolutions per minute.

a) Find the angular speed of the tire in radians per second

$\omega = \frac{\theta}{t}$

$$\frac{80\,\cancel{rev}}{1\,\cancel{min}} \cdot \frac{2\pi\text{ rad}}{1\,\cancel{rev}} \cdot \frac{1\,\cancel{min}}{60\text{ sec}} = \frac{160\pi}{60} = \frac{8\pi}{3}\text{ rad/sec}$$

b) Find the speed in inches per minute and ~~miles per minute~~

$v = r\omega$ or $v = r\frac{\theta}{t}$

$$V = (9\text{ in})\left(\frac{8\pi}{3}\text{ rad/sec}\right) = \frac{72\pi}{3} = 24\pi\text{ in/sec} = 1440\pi\text{ in/min}$$

Example: Find the angular velocity in radians per minute of a Ferris wheel 250 ft in diameter that takes 45 seconds to rotate once. Leave answer in terms of π.

$8\pi/3$ rad/min

4523.89 in/m

b. If you sat on the rim of this Ferris wheel, what would your linear velocity be?

$$V = r\omega = (125)\left(\frac{8\pi}{3}\right) = \frac{1000\pi}{3}\text{ ft/min}$$

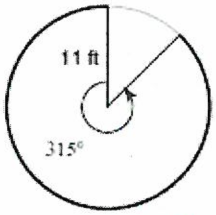
Example: A car is traveling at a speed of 45 mph. Find the angular velocity of a tire in revolutions per minute (rpm) if the diameter of ea

1047.198 ft/min

Arc Length and Sector Area

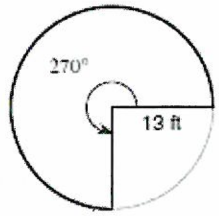
Date ~~Key~~ Key Period____

Find the length of each arc. Round your answers to the nearest tenth.

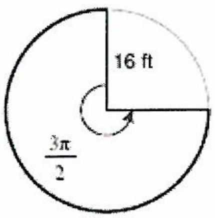
1)


19.25π ft
$= 60.5$ ft

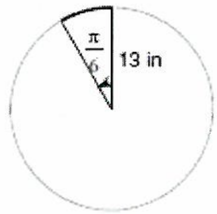
$315° \cdot \frac{\pi}{180} = \theta$

2)


$13\left(\frac{3\pi}{2}\right) = 19.5\pi$
$= 61.3$ ft

3)


24π
$= 75.4$ ft

4)


5) $r = 18$ cm, $\theta = 60°$ $18\left(\frac{\pi}{3}\right) = 6\pi$
$= 18.8$ cm

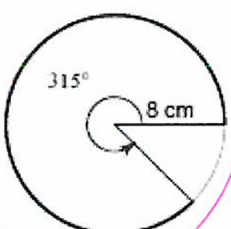
$60 \cdot \frac{\pi}{180}$

6) $r = 16$ m, $\theta = 75°$

7) $r = 9$ ft, $\theta = \frac{7\pi}{4}$ $9\left(\frac{7\pi}{4}\right) = 15.75\pi$
$= 49.5$ ft

8) $r = 14$ ft, $\theta = \frac{19\pi}{12}$

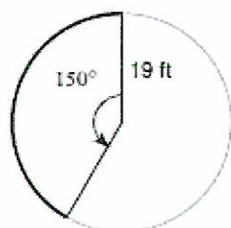
Find the length of each arc. Do not round. = Leave in terms of π

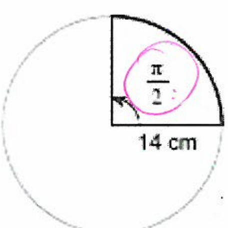
9)


14π cm

$315 \cdot \frac{\pi}{180} = \frac{7\pi}{4}$

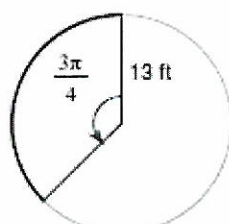
$\cancel{8}^{2}\left(\frac{7\pi}{\cancel{4}}\right)$

10)


11)


7π cm

$S = r\theta$
$= \cancel{14}_{1}\left(\frac{\pi}{\cancel{2}}\right) = 7\pi$ cm

12)


Day 2 Angles and Radian Measure Applications

Key

1. The minute hand of a clock moves from 12 to 2 o'clock, or 1/6 of a complete revolution. Through how many degrees does it move? Through how many radians does it move?

   $\frac{1}{6}(360) = 60^\circ$ $\pi/3$ radians

2. Find the distance s covered by a point moving with linear velocity $v = 55$ mi/hr and $t = 0.5$ hr.

   $d = (55)(.5) = 27.5$ mi

3. A bicycle traveled a distance of 100 meters. The diameter of the wheel of this bicycle is 40 cm. Find the number of rotations of the wheel.

   $s = 100$ m $r = 20$ cm $= .2$ m

   $100 = .2\theta$ $\theta = 500$ radians rotations $= \frac{500}{2\pi} = 79.6$ rotations

4. The wheel of a car made 100 rotations. What distance has the car traveled if the diameter of the wheel is 60 cm?

   $100 \text{ rot.} \cdot \frac{2\pi \text{ rad}}{1 \text{ rot}} = 200\pi$ rad

   $r = 30$ cm $s = r\theta = 30(200\pi) = 18849.6 \text{ cm} = 188.496$ m

5. The wheel of a machine rotates at the rate of 300 rpm (rotation per minute). If the diameter of the wheel is 80 cm, what are the angular (in radian per second) and linear speed (in cm per second) of a point on the wheel?

   a) $\frac{300 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \boxed{31.42 \text{ rad/sec}}$

   b) $v = r\omega = 40 \text{ cm}(31.42) = \boxed{1256.6 \text{ cm/sec}}$

6. The Earth rotates about its axis once every 24 hours (approximately). The radius R of the equator is approximately 4000 miles. Find the angular (radians / second) and linear (feet / second) speed of a point on the equator.

   $\frac{1 \text{ rev}}{24 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev.}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{2\pi}{86400} \text{ rad/sec}$

   b) $v = 4000\left(\frac{2\pi}{86400}\right) = .29 \text{ miles/sec} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = \boxed{1535.9 \text{ ft/s}}$ $= .0000727$ rad/sec

7. The diameter of the Ferris wheel is 250 *ft* and one complete revolution takes 20 minutes, find the linear velocity, in miles per hour, of a person riding on the wheel.

   $\frac{1 \text{ rev}}{20 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 18.85 \text{ rad/hr}$

   $v = (\cancel{250}\ 125 \text{ ft})(18.85 \text{ rad/hr})$
   $= 2356.2 \text{ ft/hr}$
   $= .445$ mph

8. Earth travels about the sun in an orbit that is almost circular. Assume that the orbit is a circle with radius 93,000,000 mi. Its angular and linear speeds are used in designing solar-power facilities.

   a. Assume that a year is 365 days, and find the angle formed by Earth's movement in one day.

   $\frac{2\pi}{365} = .0172$ rad per day

   b. Give the angular speed in radians per hour.

   $\frac{.0172 \text{ rad}}{\text{day}} \cdot \frac{1 \text{ day}}{24 \text{ hrs}} = .000717$ rad/hr

   c. Find the linear speed of Earth in miles per hour.

   $v = (93000000 \text{ mi})(.000717 \text{ rad/hr}) = 66705$ mph