Math 3   Name_____key_____

Review for Unit # 3 Test WS

Perform the indicated operation.

1) $g(a) = 2a - 4$
   $f(a) = 3a - 2$
   Find $(5g + 2f)(a)$

   $5(2a - 4) + 2(3a - 2)$
   $10a - 20 + 6a - 4$
   $= \boxed{16a - 24}$

2) $f(x) = -x + 3$
   $g(x) = x^2 + 5x$
   Find $(5f - 2g)(x)$

   $5(-x + 3) - 2(x^2 + 5x)$
   $-5x + 15 - 2x^2 - 10x$
   $= \boxed{-2x^2 - 15x + 15}$

3) $f(x) = 2x - 5$
   $g(x) = 2x + 1$
   Find $(f \cdot g)(x)$

   $(2x - 5)(2x + 1)$

	$2x$	-5
$2x$	$4x^2$	$-10x$
1	$2x$	-5

   $= \boxed{4x^2 - 8x - 5}$

4) $g(x) = 4x - 3$
   $f(x) = 2x + 1$
   Find $\left(\dfrac{g}{f}\right)(x)$

   $\boxed{\dfrac{4x-3}{2x+1}, \ x \neq -\frac{1}{2}}$

   $2x + 1 \neq 0$
   $2x \neq -1$
   $x \neq -\frac{1}{2}$

5) $f(n) = -4n - 1$
   $g(n) = 3n + 2$
   Find $(f \circ g)(n)$
   $f(g(n))$

   $-4(3n + 2) - 1$
   $= -12n - 8 - 1$
   $= \boxed{-12n - 9}$

6) $g(n) = n^3 + 2$
   $h(n) = 2n - 3$
   Find $(g \circ h)(n)$
   $g(h(n))$

   $(2n - 3)^3 + 2$
   $= 8n^3 - 36n^2 + 54n - 27 + 2$

$2n$	$4n^2 - 12n$	9
	$8n^3$	$-24n$	$16n$
-3	$-12n^2$	$36n$	-27

   $= \boxed{8n^3 - 36n^2 + 54n - 25}$

7) $g(n) = 4n - 2$
   $f(n) = n + 4$
   Find $(g \circ f)(-9)$
   $g(f(-9))$

   $f(-9) = -9 + 4 = -5$
   $g(-5) = 4(-5) - 2 = \boxed{-22}$

8) $g(t) = 2t - 4$
   $f(t) = t^2 - 2t$
   Find $(g \cdot f)(-1)$

   $(2t - 4)(t^2 - 2t)$
   $2t^3 - 4t^2 - 4t^2 + 8t$
   $2t^3 - 8t^2 + 8t$
   $2(-1)^3 - 8(-1)^2 + 8(-1) = \boxed{-18}$

9) $f(x) = 3x + 1$
   $g(x) = x^2 - 3x$
   Find $(4f + 3g)(-7)$

   $4(3x + 1) + 3(x^2 - 3x)$
   $= 12x + 4 + 3x^2 - 9x$
   $= 3x^2 + 3x + 4$
   $= 3(-7)^2 + 3(-7) + 4 = \boxed{130}$

10) $f(t) = t^2 + 3$
    $g(t) = 2t + 1$
    Find $(f \circ g)(-7)$
    $f(g(-7))$

    $g(-7) = 2(-7) + 1 = -13$
    $f(-13) = (-13)^2 + 3 = \boxed{172}$

Describe the end behavior of each function.

11) $f(x) = x^3 - 13x^2 + 56x - 82$

    $x \to -\infty, \ y \to \boxed{-\infty}$
    $x \to \infty, \ y \to \boxed{\infty}$

12) $f(x) = -x^4 - x^3 + x^2 - 1$

    $x \to -\infty, \ y \to \boxed{-\infty}$
    $x \to \infty, \ y \to \boxed{-\infty}$

13) $f(x) = -x^5 + 4x^3 - 4x + 1$

    $x \to -\infty, \ y \to \boxed{\infty}$
    $x \to \infty, \ y \to \boxed{-\infty}$

14) $f(x) = x^2 - 4x - 2$

    $x \to -\infty, \ y \to \boxed{\infty}$
    $x \to \infty, \ y \to \boxed{\infty}$

Write a polynomial function using the given zeros. work on sep. sheet

15) $-3, \ -4, \ 2$

    $\boxed{P(x) = x^3 + 5x^2 - 2x - 24}$

16) -3 mult. 2, $-\dfrac{4}{5}$

    $\boxed{P(x) = 5x^3 + 34x^2 + 69x + 36}$

17) $\dfrac{1}{3}, \ \dfrac{3}{2}, \ 0, \ \dfrac{1}{2}$

    $\boxed{P(x) = 12x^4 - 28x^3 + 17x^2 - 3x}$

18) $-\dfrac{1}{2}$ mult. 2, $-\dfrac{3}{2}, \ 5$

    $\boxed{P(x) = 8x^4 - 20x^3 - 86x^2 - 67x - 15}$

Graph each polynomial function very ACCURATELY.

19) $f(x) = x^3 - 3x^2 + 4$
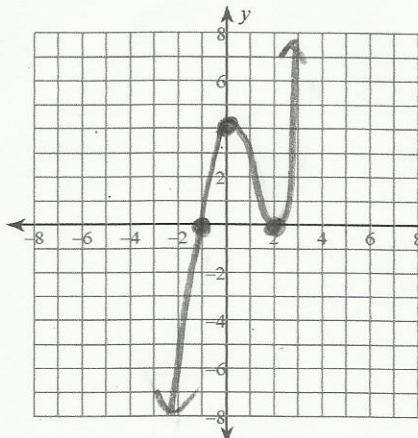
real zeros = -1,
2(mo2)

Max = (2,0)

Min = (0,4)

Yint = (0,4)

20) $f(x) = x^4 + 3x^3 - 5 - 5x$
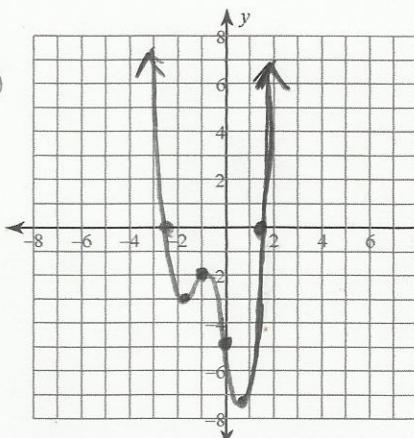
real zeros = -2.5,
1.4

Min = (-1.9, -3)
    (0.7, -7.2)

Max = (-1, 2)

Yint = (0, -5)

Divide using long or synthetic division (which ever method is appropriate). work on sep sheet.

21) $(18p^3 + 6p^2 - 76p + 47) \div (6p - 8)$

$$3p^2 + 5p - 6 - \frac{1}{6p-8}$$

22) $(3n^4 + 13n^3 + 23n^2 + 36n + 38) \div (3n + 7)$

$$n^3 + 2n^2 + 3n + 5 + \frac{3}{3n+7}$$

23) $(a^4 + 3a^3 - 44a^2 + 27a - 44) \div (a - 5)$

$$a^3 + 8a^2 - 4a + 7 - \frac{9}{a-5}$$

24) $(6n^5 + 59n^4 + 45n^3 + 8n + 75) \div (n + 9)$

$$6n^4 + 5n^3 + 8 + \frac{3}{n+9}$$

Using factoring and other quadratic techniques, find all zeros for each polynomial.

25) $(x^3 - 4x^2 - 2x + 8) = 0$

$x^2 - 2 = 0$ $x - 4 = 0$

$x^2(x-4) - 2(x-4) = 0$ $x^2 = 2$ $\boxed{x = 4}$

$(x^2 - 2)(x-4) = 0$ $\boxed{x = \pm\sqrt{2}}$

26) $x^4 + 9x^2 + 20 = 0$

$x^2 + 5 = 0$ $x^2 + 4 = 0$

$(x^2 + 5)(x^2 + 4) = 0$

$x^2 = -5$ $x^2 = -4$

$\boxed{x = \pm i\sqrt{5}}$ $\boxed{x = \pm 2i}$

27) $x^3 + 64 = 0$

$(x+4)(x^2 - 4x + 16) = 0$ $\dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$

$x + 4 = 0$

$\boxed{x = -4}$ $\dfrac{4 \pm \sqrt{-48}}{2} = \dfrac{4 \pm 4i\sqrt{3}}{2} = \boxed{2 \pm 2i\sqrt{3}}$

28) $x^5 + 2x^3 - 3x = 0$

$x(x^4 + 2x^2 - 3) = 0$

$x(x^2 + 3)(x^2 - 1) = 0$ $x^2 - 1 = 0$

$\boxed{x = 0}$ $x^2 + 3 = 0$ $x^2 = 1$

$x^2 = -3$ $\boxed{x = \pm 1}$

$\boxed{x = \pm i\sqrt{3}}$

Find the remaining zero(s) for each polynomial and its given information. Work on sep. sheet.

29) $f(x) = 2x^3 + x^2 - 14x + 12$

where $(2x - 3)$ is a factor.

$$\boxed{-1 \pm \sqrt{5}}$$

30) $f(x) = 3x^3 - 19x^2 + 28x$

where x and $(x - 4)$ are factors.

$$\boxed{x = \frac{7}{3}}$$

31) $f(x) = 2x^4 + 8x^3 - 7x^2 - 42x - 9$

where $(x + 3)^2$ are factors.

$$\boxed{\frac{2 \pm \sqrt{6}}{2}}$$

32) $f(x) = 60x^4 + 61x^3 - 155x^2 - 62x + 24$

where $(4x - 1)$, $(x + 2)$, and $(3x - 4)$ are factors.

$$\boxed{x = -\frac{3}{5}}$$

Find all zeros for each polynomial.

33) $f(x) = 2x^3 + 7x^2 + 2x - 6$ $RZ = -\frac{3}{2}$

$-\frac{3}{2}\big|\ \ 2\ \ \ 7\ \ \ 2\ \ -6$
$\underline{\ \ \ \ \ \ \ -3\ -6\ \ \ 6}$
$\ \ \ \ \ 2\ \ \ 4\ -4\ \big|\ 0$

$2x^2 + 4x - 4 = 0$
$x^2 + 2x - 2 = 0$

$\dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$

$\dfrac{2 \pm \sqrt{12}}{2}$

$\dfrac{-2 \pm 2\sqrt{3}}{2}$

$= -1 \pm \sqrt{3}$

$\boxed{\text{all zeros} = -\frac{3}{2}, -1 \pm \sqrt{3}}$

34) $f(x) = 7x^3 - 11x^2 - 7x + 11$ $RZ = \pm 1$

$1\big|\ \ 7\ -11\ -7\ \ \ 11$
$\underline{\ \ \ \ \ \ \ \ \ 7\ -4\ -11}$
$-1\big|\ 7\ \ -4\ -11\ \big|\ 0$
$\underline{\ \ \ \ \ \ \ \ \ -7\ \ \ \ \ 11}$
$\ \ \ \ \ 7\ -11\ \big|\ 0$

$7x - 11 = 0$
$7x = 11$
$x = \frac{11}{7}$

$\boxed{\text{all zeros} = \pm 1, \frac{11}{7}}$

35) $f(x) = 3x^4 + 4x^3 + 8x^2$ $RZ = 0\,(mo2)$

$0\big|\ \ 3\ \ \ 4\ \ \ 8\ \ \ 0\ \ \ 0$
$\underline{\ \ \ \ \ \ 0\ \ \ 0\ \ \ 0\ \ \ 0}$
$0\big|\ \ 3\ \ \ 4\ \ \ 8\ \ \ 0\ \big|\ 0$
$\underline{\ \ \ \ \ \ 0\ \ \ 0\ \ \ 0}$
$\ \ \ \ \ 3\ \ \ 4\ \ \ 8\ \big|\ 0$

$3x^2 + 4x + 8 = 0$

$\dfrac{-4 \pm \sqrt{(4)^2 - 4(3)(8)}}{2(3)} = \dfrac{-4 \pm \sqrt{-80}}{6}$

$\dfrac{-4 \pm 4i\sqrt{5}}{6}$

$= \dfrac{-2 \pm 2i\sqrt{5}}{3}$

$\boxed{\text{all zeros} = 0\,(mo2), \dfrac{-2 \pm 2i\sqrt{5}}{3}}$

36) $f(x) = 3x^4 - 7x^3 + 42x^2 - 112x - 96$ $RZ = -\frac{2}{3}, 3$

$-\frac{2}{3}\big|\ \ 3\ -7\ \ \ 42\ -112\ -96$
$\underline{\ \ \ \ \ \ \ \ -2\ \ \ \ 6\ \ \ -32\ \ \ \ 96}$
$3\big|\ \ 3\ -9\ \ \ 48\ -144\ \big|\ 0$
$\underline{\ \ \ \ \ \ \ \ 9\ \ \ \ 0\ \ \ \ 144}$
$\ \ \ \ \ 3\ \ \ 0\ \ \ 48\ \big|\ 0$

$3x^2 + 48 = 0$
$3x^2 = -48$
$x^2 = -16$
$x = \pm 4i$

$\boxed{\text{all zeros} = -\frac{2}{3}, 3, \pm 4i}$

37) $f(x) = 6x^5 + 37x^4 - 4x^3 - 264x^2 - 128x + 128$

$RZ = -4\,(mo2), \frac{1}{2}$

$-4\big|\ \ 6\ \ \ 37\ \ \ -4\ \ \ -264\ \ \ -128\ \ \ 128$
$\underline{\ \ \ \ \ \ -24\ -52\ \ \ 224\ \ \ \ 160\ \ \ -128}$
$-4\big|\ \ 6\ \ \ 13\ \ -56\ \ \ -40\ \ \ \ \ 32\ \big|\ 0$
$\underline{\ \ \ \ \ \ -24\ \ \ 44\ \ \ \ 48\ \ \ -32}$
$\frac{1}{2}\big|\ \ 6\ -11\ -12\ \ \ \ 8\ \big|\ 0$
$\underline{\ \ \ \ \ \ \ \ 3\ \ -4\ \ \ -8}$
$\ \ \ \ \ 6\ \ -8\ -16\ \big|\ 0$

$6x^2 - 8x - 16 = 0$
$3x^2 - 4x - 8 = 0$

$\dfrac{4 \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)} = \dfrac{4 \pm \sqrt{112}}{6}$

$\dfrac{4 \pm 4\sqrt{7}}{6}$

$\dfrac{2 \pm 2\sqrt{7}}{3}$

$\boxed{\text{all zeros} = -4\,(mo2), \frac{1}{2}, \dfrac{2 \pm 2\sqrt{7}}{3}}$

38) $f(x) = 24x^5 - 26x^4 - 205x^3 + 319x^2 + 96x - 180$

$RZ = 2\,(mo2), -3, \frac{3}{4}$

$2\big|\ \ 24\ -26\ \ -205\ \ \ 319\ \ \ 96\ \ -180$
$\underline{\ \ \ \ \ \ \ \ \ 48\ \ \ \ 44\ \ -322\ \ \ -6\ \ \ 180}$
$2\big|\ \ 24\ \ \ 22\ \ -161\ \ \ -3\ \ \ 90\ \big|\ 0$
$\underline{\ \ \ \ \ \ \ \ \ 48\ \ \ 140\ \ -42\ \ -90}$
$-3\big|\ \ 24\ \ \ 70\ \ \ -21\ \ -45\ \big|\ 0$
$\underline{\ \ \ \ \ \ \ \ -72\ \ \ \ 6\ \ \ \ 45}$
$-\frac{3}{4}\big|\ \ 24\ \ -2\ \ -15\ \big|\ 0$
$\underline{\ \ \ \ \ \ \ \ -18\ \ \ 15}$
$\ \ \ \ \ 24\ \ \ 20\ \big|\ 0$

$24x - 20 = 0$
$\dfrac{24x}{24} = \dfrac{20}{24}$
$x = \dfrac{5}{6}$

$\boxed{\text{all zeros} = 2\,(mo2), -3, \frac{3}{4}, \frac{5}{6}}$

Math 3 — Review for Unit #3 Test

15) $(x+3)(x+4)(x-2)$

$(x^2+7x+12)(x-2)$

	x^2	$7x$	12
x	x^3	$7x^2$	$12x$
-2	$-2x^2$	$-14x$	-24

$$\boxed{P(x)=x^3+5x^2-2x-24}$$

16) $(x+3)(x+3)(5x+4)$

$(x^2+6x+9)(5x+4)$

	x^2	$6x$	9
$5x$	$5x^3$	$30x^2$	$45x$
4	$4x^2$	$24x$	36

$$\boxed{P(x)=5x^3+34x^2+69x+36}$$

17) $(3x-1)(2x-3)(2x-1)x$

$(6x^2-11x+3)(2x^2-x)$

	$6x^2$	$-11x$	3
$2x^2$	$12x^4$	$-22x^3$	$6x^2$
$-x$	$-6x^3$	$11x^2$	$-3x$

$$\boxed{P(x)=12x^4-28x^3+17x^2-3x}$$

18) $(2x+1)(2x+1)(2x+3)(x-5)$

$(4x^2+4x+1)(2x^2-7x-15)$

	$4x^2$	$4x$	1
$2x^2$	$8x^4$	$8x^3$	$2x^2$
$-7x$	$-28x^3$	$-28x^2$	$-7x$
-15	$-60x^2$	$-60x$	-15

$$\boxed{P(x)=8x^4-20x^3-86x^2-67x-15}$$

21)

$$3p^2+5p-6-\dfrac{1}{6p-8}$$

$6p-8\,\overline{)\,18p^3+6p^2-76p+47}$

$\quad -18p^3+24$

$\qquad 30p^2-76p$

$\qquad -30p^2+40p$

$\qquad\qquad -36p+47$

$\qquad\qquad +36p-48$

$\qquad\qquad\qquad -1$

22)

$$n^3+2n^2+3n+5+\dfrac{3}{3n+7}$$

$3n+7\,\overline{)\,3n^4+13n^3+23n^2+36n+38}$

$\quad -3n^4+7n^3$

$\qquad -6n^3+23n^2$

$\qquad -6n^3+14n^2$

$\qquad\qquad 9n^2+36$

$\qquad\qquad -9n^2+21n$

$\qquad\qquad\qquad 15n+38$

$\qquad\qquad\qquad -15n+35$

$\qquad\qquad\qquad\qquad 3$

23)

$$
\begin{array}{r|rrrr}
5 & 1 & -44 & 27 & -44 \\
  &   & 5 & 40 & 70 & 35 \\
\hline
  & 1 & 8 & -4 & 7 & | -9
\end{array}
$$

$$\boxed{a^3+8a^2-4a+7-\dfrac{9}{a-5}}$$

24)

$$
\begin{array}{r|rrrrrr}
-9 & 6 & 59 & 45 & 0 & 8 & 75 \\
   &   & -54 & -45 & 0 & 0 & -72 \\
\hline
   & 6 & 5 & 0 & 0 & 8 & | 3
\end{array}
$$

$$\boxed{6n^4+5n^3+8+\dfrac{3}{n+9}}$$

29)

$$\frac{3}{2} \,\big|\; 2 \quad 1 \quad -14 \quad 12$$
$$\downarrow \quad 3 \quad 6 \quad -12$$
$$\overline{\quad 2 \quad 4 \quad -8 \;\big|\; 0\quad}$$

$$2x^2 + 4x - 8 = 0$$
$$x^2 + 2x - 4 = 0$$
$$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$
$$\frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2}$$
$$= \boxed{-1 \pm \sqrt{5}}$$

30)

$$0 \,\big|\; 3 \quad -19 \quad 28 \quad 0$$
$$\downarrow \quad 0 \quad 0 \quad 0$$
$$4 \,\big|\; \overline{3 \quad -19 \quad 28 \;\big|\; 0\quad}$$
$$\downarrow \quad 12 \quad -28$$
$$\overline{\quad 3 \quad -7 \;\big|\; 0\quad}$$

$$3x - 7 = 0$$
$$3x = 7$$
$$\boxed{x = \frac{7}{3}}$$

31)

$$-3 \,\big|\; 2 \quad 8 \quad -7 \quad -42 \quad -9$$
$$\downarrow \quad -6 \quad -6 \quad 39 \quad 9$$
$$-3 \,\big|\; \overline{2 \quad 2 \quad -13 \quad -3 \;\big|\; 0\quad}$$
$$\downarrow \quad -6 \quad 12 \quad 3$$
$$\overline{\quad 2 \quad -4 \quad -1 \;\big|\; 0\quad}$$

$$2x^2 - 4x - 1 = 0$$
$$\frac{4 \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)} = \frac{4 \pm \sqrt{24}}{4}$$
$$= \frac{4 \pm 2\sqrt{6}}{4} = \boxed{\frac{2 \pm \sqrt{6}}{2}}$$

32)

$$\frac{1}{4} \,\big|\; 60 \quad 61 \quad -155 \quad -62 \quad 24$$
$$\downarrow \quad 15 \quad 19 \quad -34 \quad 24$$
$$-2 \,\big|\; \overline{60 \quad 76 \quad -136 \quad -96 \;\big|\; 0\quad}$$
$$\downarrow \quad -120 \quad 88 \quad 96$$
$$\frac{4}{3} \,\big|\; \overline{60 \quad -44 \quad -48 \;\big|\; 0\quad}$$
$$\downarrow \quad 80 \quad 48$$
$$\overline{\quad 60 \quad 36 \;\big|\; 0\quad}$$

$$60x + 36 = 0$$
$$\frac{60x = -36}{60 \qquad 60}$$
$$\boxed{x = -\frac{3}{5}}$$