Convergence Tests $\Rightarrow$ tell if series conv. or div. but won't find sum

Integral Test

If $\sum_{n=1}^{\infty} a_n$ is a positive series, let $f(n) = a_n$ and f be the function obtained by replacing n with x. If $f(x)$ is positive, continuous, and decreasing for $x \geq 1$, then

$\sum_{n=1}^{\infty} a_n$

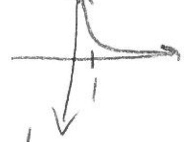
a) converges if $\int_1^{\infty} f(x)\,dx$ converges

b) diverges if $\int_1^{\infty} f(x)\,dx$ diverges

* The value you get for $\int_1^{\infty} f(x)\,dx \neq \sum_{n=1}^{\infty} a_n$

* Must satisfy 3 conditions $\Rightarrow$ positive, continuous, decreasing $x \geq 1$.

Ex. $\underset{\text{Harmonic}}{\sum_{n=1}^{\infty} \frac{1}{n}}$ Let $f(x) = \frac{1}{x}$. When $x \geq 1$, $f(x)$ is pos & cont.



$f(x)$ decr.

$f'(x) = \frac{-1}{x^2} < 0$ when $x \geq 1$

so $f(x)$ decreases.

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b \to \infty} \int_1^{b} \frac{1}{x}\,dx = \lim_{b \to \infty} \left[\ln|x| \Big|_1^b \right]$$

$$= \lim_{b \to \infty} \left[\ln|b| - \underbrace{\ln 1}_{0} \right] = \infty \text{ diverges}$$

$\therefore$ By integral test, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

$\sum_{n=1}^{\infty} ne^{-n^2}$ Conv. or div. & why?

Let $f(x)=xe^{-x^2}$. For $x \geq 1$, $f(x)$ is pos. + cont.

$f'(x)=e^{-x^2}+x(-2x)e^{-x^2} = e^{-x^2}(1-2x^2)<0$ when $x \geq 1$

so $f(x)$ decreasing.

$$\int_1^{\infty} xe^{-x^2}dx = \lim_{b\to\infty}\int_1^b xe^{-x^2}dx = \lim_{b\to\infty}\left[-\frac{1}{2}e^{-x^2}\Big|_1^b\right]$$

$$= \lim_{b\to\infty}\left[\underbrace{-\frac{1}{2}e^{-b^2}}_{0}+\frac{1}{2}e^{-1}\right] = \frac{1}{2e} \text{ converges}$$

$\therefore$ By Integral test, $\sum_{n=1}^{\infty} ne^{-n^2}$ converges.

P series $\Rightarrow$ Hyper-harmonic series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ where p is pos. real #.

if $p>1$, converges

if $p\leq 1$, diverges

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ p-series $p=2>1$ so converges

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ p series $p=\frac{1}{2}\leq 1$ so diverges