8.9 Power Rep. of Functions

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$$

Geometric form of infinite sum $\Rightarrow f(x) = \dfrac{1^{st}\text{ term}}{1-r}$

Ex. $f(x) = \dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots x^n + \cdots = \sum_{n=0}^{\infty} x^n$

(1^{st} term: 1; r: x)

IOC: $|x| < 1 \Rightarrow (-1, 1)$

Geo $\Rightarrow \sum_{n=0}^{\infty} a_0 r^n$

Ex. $f(x) = \dfrac{x}{1+x} = x - x^2 + x^3 - x^4 + \cdots x^1(-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{n+1}$

(1^{st} term: x; $r = -x$)

Ex. $f(x) = \dfrac{3}{1-2x} = 3 + 6x + 12x^2 + \cdots 3(2x)^n + \cdots = \sum_{n=0}^{\infty} 3 \cdot 2^n x^n$

(1^{st} term: 3; r: $2x$)

IOC: $|2x| < 1 \Rightarrow |x| < \frac{1}{2}$

$\left(-\frac{1}{2}, \frac{1}{2}\right)$

$= 3\sum_{n=0}^{\infty} 2^n x^n$

Ex Find power series centered at 0 for $f(x) = \dfrac{4}{2+3x}$

$$f(x) \overset{\cdot \frac{1}{2}}{\underset{\cdot \frac{1}{2}}{}} \frac{4}{2+3x} = \frac{2}{1+\frac{3}{2}x} = 2 - 3x + \frac{9}{2}x^2 - \frac{27}{4}x^3 + \cdots + 2\left(-\frac{3}{2}x\right)^n + \cdots$$

(1^{st} term: 2; $r = -\frac{3}{2}x$)

$$= \sum_{n=0}^{\infty} \frac{(-3)^n}{2^{n-1}} x^n$$

Recentering

Ex. $f(x) = \frac{2}{1-x}$ centered at $x=-2$

$$= \frac{\frac{1}{3}\cdot 2}{\frac{1}{3}3-\frac{1}{3}(x+2)} = \frac{\frac{2}{3}}{1-\frac{1}{3}(x+2)} \quad \leftarrow 1^{st} \text{ term}$$

$$= \frac{2}{3} + \frac{2}{9}(x+2) + \frac{2}{27}(x+2)^2 + \cdots \left(\frac{2}{3}\right)\left[\frac{1}{3}(x+2)\right]^n$$

$$= \sum_{n=0}^{\infty} \frac{2}{3^{n+1}}(x+2)^n$$

HW. p623 # 35-37 p630 # 5-12, 19, 21, 23