AB Practice Exam 1 (AB Final Review)

No Calculator Part

1. If $f'(x) = \ln(x-2)$, then the graph of $y = f(x)$ is decreasing if and only if

(A) $2 < x < 3$ (B) $0 < x$ (C) $0 < x < 1$ (D) $x > 1$ (E) $x > 2$

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2. For $x \neq 0$, the slope of the tangent to $y = x\cos x$ equals zero whenever

(A) $\tan x = -x$

(B) $\tan x = \dfrac{1}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

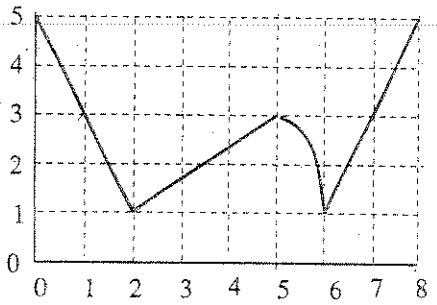
(E) $\cos x = x$

3. The function F is defined by

$F(x) = G[x + G(x)]$

where the graph of the function G is shown at the right.

graph of the function G



The approximate value of $F'(1)$ is

(A) $\frac{7}{3}$

(B) $\frac{2}{3}$

(C) -2

(D) -1

(E) $-\frac{2}{3}$

4. $\displaystyle\int_2^6 \left(\frac{1}{x} + 2x\right) dx =$

(A) $\ln 4 + 32$

(B) $\ln 3 + 40$

(C) $\ln 3 + 32$

(D) $\ln 4 + 40$

(E) $\ln 12 + 32$

5. A relative maximum of the function $f(x) = \dfrac{(\ln x)^2}{x}$ occurs at

(A) 0

(B) 1

(C) 2

(D) e

(E) e^2

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6. Use a right-hand Riemann sum with 4 equal subdivisions to approximate the integral $\displaystyle\int_{-1}^{3} |2x - 3|\, dx$.

(A) 13

(B) 10

(C) 8.5

(D) 8

(E) 6

7. An equation of the line tangent to the graph of $y = x^3 - 3x^2 + 2$ at its point of inflection is

(A) $y = -3x + 1$

(B) $y = -3x - 7$

(C) $y = x + 5$

(D) $y = 3x + 1$

(E) $y = 3x + 7$

8. $\int \cos(3 - 2x)\, dx =$

(A) $\sin(3 - 2x) + C$

(B) $-\sin(3 - 2x) + C$

(C) $\frac{1}{2}\sin(3 - 2x) + C$

(D) $-\frac{1}{2}\sin(3 - 2x) + C$

(E) $-\frac{1}{5}\sin(3 - 2x) + C$

9. What is $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 2}}{4x + 3}$?

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{2}}{3}$ (D) 1 (E) The limit does not exist.

10. Let the first quadrant region enclosed by the graph of $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the x-axis are semicircles, the volume of the solid is

(A) $\frac{3\pi}{64}$ units^3

(B) $\frac{3\pi}{32}$ units^3

(C) $\frac{3\pi}{16}$ units^3

(D) $\frac{3\pi}{8}$ units^3

(E) $\frac{3\pi}{4}$ units^3

11. Let $f(x) = \ln x + e^{-x}$. Which of the following is TRUE at $x = 1$?

(A) f is increasing

(B) f is decreasing

(C) f is discontinuous

(D) f has a relative minimum

(E) f has a relative maximum

12. Suppose $F(x) = \int_0^{x^2} \frac{1}{2 + t^3}\, dt$ for all real x, then $F'(-1) =$

(A) 2 (B) 1 C) $\frac{1}{3}$ (D) –2 (E) $-\frac{2}{3}$

13. What is the average (mean) value of $2t^3 - 3t^2 + 4$ over the interval $-1 \le t \le 1$?

(A) 0

(B) $\frac{7}{4}$

(C) 3

(D) 4

(E) 6

14. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = x^2 + k$ for every real value of k?

(A) $y = -x^2$

(B) $y = -\dfrac{1}{x^2}$

(C) $y = \dfrac{1}{x^2}$

(D) $y = -\dfrac{1}{2}\ln x$

(E) $y = \ln x$

15. What is $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The limit does not exist.

16. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1

(B) 0

(C) $-2(\cos x + \sin x)$

(D) $2(\cos x + \sin x)$

(E) $-4(\cos x)(\sin x)$

17. The area under the graph of $y = 4x^3 + 6x - \dfrac{1}{x}$ on the interval $1 \le x \le 2$ is

(A) $32 - \ln 2$ units^2

(B) $30 - \ln 2$ units^2

(C) $24 - \ln 2$ units^2

(D) $\frac{99}{4}$ units^2

(E) 21 units^2

18. $\displaystyle\int \frac{x-2}{x-1}\,dx =$

(A) $-\ln|x-1| + C$

(B) $x + \ln|x-1| + C$

(C) $x - \ln|x-1| + C$

(D) $x + \sqrt{x-1} + C$

(E) $x - \sqrt{x-1} + C$

19. Suppose that g is a function with the following two properties: $g(-x) = g(x)$ for all x, and $g'(a)$ exists. Which of the following must necessarily be equal to $g'(-a)$?

(A) $g'(a)$ (B) $-g'(a)$ (C) $\dfrac{1}{g'(a)}$ (D) $-\dfrac{1}{g'(a)}$ (E) none

20. An equation for a tangent to the graph of $y = \text{Arctan}\,\frac{x}{3}$ at the origin is:

(A) $x - 3y = 0$

(B) $x - y = 0$

(C) $x = 0$

(D) $y = 0$

(E) $3x - y = 0$

21. If $f(x) = \begin{cases} x^2 + 4 \text{ for } 0 \le x \le 1 \\ 6 - x \text{ elsewhere} \end{cases}$ then $\int_0^3 f(x)\,dx$ is a number between

(A) 0 and 5

(B) 5 and 10

(C) 10 and 15

(D) 15 and 20

(E) 20 and 25

22. $\frac{d}{dx}\left(\ln e^{3x}\right) =$

(A) 1

(B) 3

(C) $3x$

(D) $\frac{1}{e^{3x}}$

(E) $\frac{3}{e^{3x}}$

23. If $g'(x) = 2g(x)$ and $g(-1) = 1$, then $g(x) =$

(A) e^{2x}

(B) e^{-x}

(C) e^{x+1}

(D) e^{2x+2}

(E) e^{2x-2}

24. The acceleration at time $t > 0$ of a particle moving along the x-axis is $a(t) = 3t + 2$ ft/sec^2. If at $t = 1$ seconds the velocity is 4 ft/sec and the position is $x = 6$ feet, then at $t = 2$ seconds the position $x(t)$ is

(A) 8 ft (B) 11 ft (C) 12 ft (D) 13 ft (E) 15 ft

25. The approximate value of $y = \sqrt{3 + e^x}$ at $x = 0.08$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.01

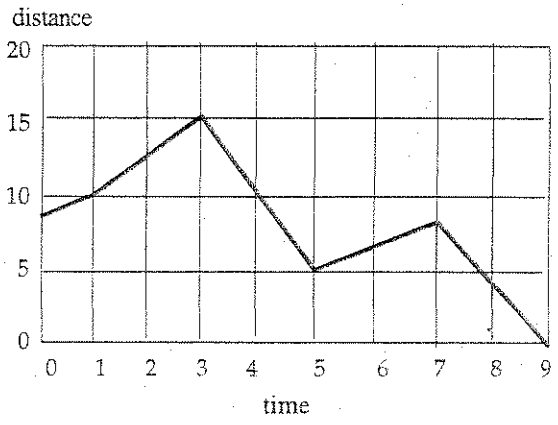
(B) 2.02

(C) 2.03

(D) 2.04

(E) 2.05

26. A leaf falls from a tree into a swirling wind. The graph at the right shows the vertical distance (feet) above the ground plotted against time (seconds).



According to the graph, in what time interval is the speed of the leaf the greatest?

(A) $1 < t < 3$

(B) $3 < t < 5$

(C) $5 < t < 7$

(D) $7 < t < 9$

(E) none of these

27. Water is flowing into a spherical tank with 6 foot radius at the constant rate of 30π cu ft per hour. When the water is h feet deep, the volume of water in the tank is given by

$$V = \frac{\pi h^2}{3}(18 - h).$$

What is the rate at which the depth of the water in the tank is increasing at the moment when the water is 2 feet deep?

(A) 0.5 ft per hr

(B) 1.0 ft per hr

(C) 1.5 ft per hr

(D) 2.0 ft per hr

(E) 2.5 ft per hr

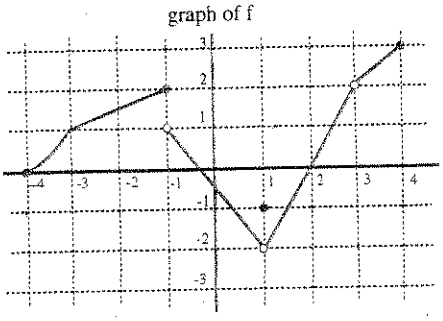
28. The graph of the function $f(x) = 2x^{5/3} - 5x^{2/3}$ is increasing on which of the following intervals.

I. $1 < x$ II. $0 < x < 1$ III. $x < 0$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

CALCULATOR PART

1. The function f is defined on the interval $[-5, 5]$ and its graph is shown to the right. Which of the following statements are true?



I. $\lim_{x \to 1} f(x) = -1$

II. $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 2$

III. $\lim_{x \to -1^-} f(x) = f(-3)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

2. For $f(x) = \sin^2 x$ and $g(x) = 0.5x^2$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the instantaneous rate of change of f is greater than the instantaneous rate of change of g for which value of x?

(A) -0.8 (B) 0 (C) 0.9 (D) 1.2 (E) 1.5

3. If $f(x) = 2x^2 - x^3$ and $g(x) = x^2 - 2x$, for what values of a and b is

$$\int_a^b f(x)\,dx > \int_a^b g(x)\,dx \ ?$$

I. $a = -1$ and $b = 0$ II. $a = 0$ and $b = 2$ III. $a = 2$ and $b = 3$

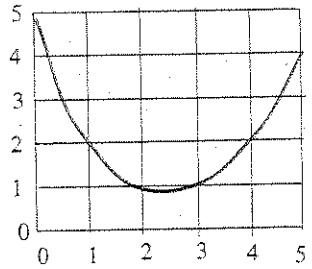
(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, III

4. If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$

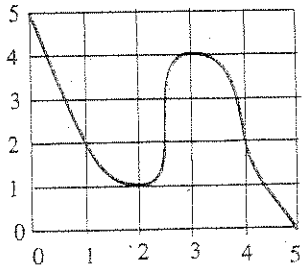
(A) $\frac{-6}{7y^3}$ (B) $\frac{-3}{y^3}$ (C) 3 (D) $\frac{3}{2y}$ (E) $\frac{-9}{4y^3}$

5. The graphs of functions f and g are shown at the right.

graph of f



graph of g



If $h(x) = g[f(x)]$, which of the following statements are true about the function h?

I. $h(0) = 4$.

II. h is increasing at $x = 2$.

III. The graph of h has a horizontal tangent at $x = 4$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

6. The minimum distance from the origin to the curve $y = e^x$ is

(A) 0.72 (B) 0.74 (C) 0.76 (D) 0.78 (E) 0.80

7. The area of the first quadrant region bounded by the y-axis, the line $y = 4 - x$ and the graph of $y = x - \cos x$ is approximately

(A) 4.50 units^2 (B) 4.54 units^2 (C) 4.56 units^2 (D) 4.58 units^2 (E) 5.00 units^2

8. The number of inflection points for the graph of $y = 2x + \cos(x^2)$ in the interval $0 \le x \le 5$ is

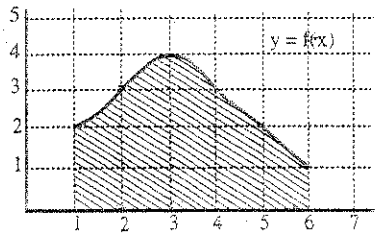
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

9. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1 + 2^t}$, where V is the volume of ice in cubic feet, and t is the time in minutes. What amount of ice has melted in the first 5 minutes?

(A) 14.49 ft^3 (B) 14.51 ft^3 (C) 14.53 ft^3 (D) 14.55 ft^3 (E) 14.57 ft^3

10. The region shaded in the figure at the right is rotated about the x-axis. Using the Trapezoid Rule with 5 equal subdivisions, the approximate volume of the resulting solid is

graph of $y = f(x)$



(A) 23 units^3

(B) 47 units^3

(C) 127 units^3

(D) 254 units^3

(E) 400 units^3

11. A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = (t+1)(t-3)^3$. For what values of t is the velocity of the particle increasing?

(A) all t (B) $0 < t < 1$ C) $0 < t < 3$ (D) $1 < t < 3$ E) $t < 1$ or $t > 3$

12. Let $f(x) = \dfrac{\ln e^{2x}}{x-1}$ for $x > 1$. If g is the inverse of f, then $g'(3) =$

(A) 2 (B) 1 (C) 0 (D) –1 (E) –2

13. $\displaystyle\int \frac{e^{x^2} - 2x}{e^{x^2}}\,dx$

(A) $x - e^{x^2} + C$

(B) $x - e^{-x^2} + C$

(C) $x + e^{-x^2} + C$

(D) $-e^{x^2} + C$

(E) $e^{-x^2} + C$

14. How many critical points does the function $f(x) = (x+2)^5(x^2-1)^4$ have?

(A) 2 (B) 3 (C) 4 D) 5 (E) 9

15. Let m and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \leq 1 \\ mx + b & \text{for } x > 1. \end{cases}$$

If f is both continuous and differentiable at $x = 1$, then

(A) $m = 1, b = 1$

(B) $m = 1, b = -1$

(C) $m = -1, b = 1$

(D) $m = -1, b = -1$

(E) none of the above

16. Suppose a car is moving with increasing speed according to the following table.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

The closest approximation of the distance traveled in the first 10 seconds is

(A) 150 ft

(B) 250 ft

(C) 350 ft

(D) 450 ft

(E) 550 ft

17. Consider the function F defined so that $F(x) + 5 = \displaystyle\int_2^x \sin\left(\frac{\pi t}{4}\right) dt$.

The value of $F(2) + F'(2)$ is

(A) 0

(B) 1

(C) $\frac{\pi}{4}$

(D) 4

(E) -4