__Alt series test__ (AST) $\Rightarrow \sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n-1} a_n$

If have alt series, $\sum_{n=1}^{\infty}(-1)^n a_n$ $\qquad \sum_{n=1}^{\infty}(\cos \pi n) a_n = \sum_{n=1}^{\infty}(-1)^n a_n$

Converges if

① $\lim_{n\to\infty} a_n = 0$

② $a_{n+1} \le a_n \Rightarrow a_n$ is __decreasing__

* Alt series test does __not__ prove divergence.

* If cond #1 is not true $\Rightarrow \lim_{n\to\infty} a_n \neq 0$ diverges by n^{th} term test.

Ex. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n}{4n^2-3}$ Conv. or div & why?

__AST__ ① $\lim_{n\to\infty}\frac{2n}{4n^2-3} = 0$ ✓

② decr? $f'(n) = \frac{2(4n^2-3)-(2n)(8n)}{(4n^2-3)^2} = \frac{-8n^2-6}{(4n^2-3)^2} < 0$

so a_n is decreasing

$\therefore$ By alt series test, $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n}{4n^2-3}$ converges.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n}{4n-3}$$

$$\lim_{n\to\infty} \frac{2n}{4n-3} = \frac{1}{2} \neq 0$$ so diverges by n^{th} term test.

Absolute convergence vs. Conditional convergence

A series converges absolutely if $\sum_{n=1}^{\infty} |a_n|$ converges.

A series converges conditionally if $\sum_{n=1}^{\infty} |a_n|$ diverges but $\sum_{n=1}^{\infty} a_n$ converges.

have to show 2 things!

If series $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

Ex $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ Abs conv? Cond conv? or div.?

Abs conv? $\sum_{n=1}^{\infty} \left| (-1)^n \frac{1}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$ harmonic so diverges

so $\sum_{n=1}^{\infty} a_n$ is not abs. conv.

Cond conv? $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

① $\lim_{n\to\infty} \frac{1}{n} = 0$

② dec? $f'(n) = -\frac{1}{n^2} < 0$ so a_n decr.

$\therefore$ By AST $\sum (-1)^n \frac{1}{n}$ conv.

So conditionally conv.