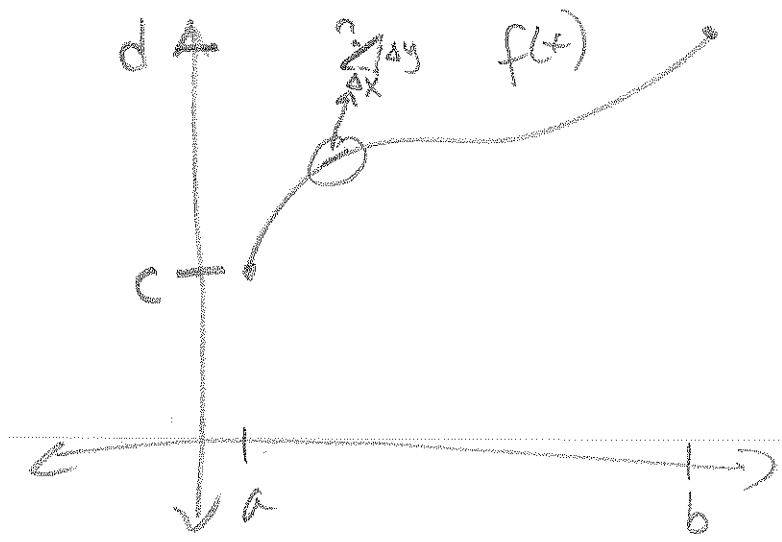
Arc length



length $\Rightarrow \sqrt{(\Delta x)^2+(\Delta y)^2}\cdot\frac{\Delta x}{\Delta x}$

$$\sqrt{\frac{(\Delta x)^2+(\Delta y)^2}{(\Delta x)^2}}\,\Delta x$$

$$\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$$

$$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Ex. Find the length of $f(x)=x^3-2x^2+1$ on $[-1,3]$.

$$L=\int_{-1}^{3}\sqrt{1+\underbrace{(3x^2-4x)}_{y'}{}^2}\,dx$$

$$=15.603$$

$$=\int_a^b \sqrt{1+(f'(x))^2}\,dx$$

$$=\int_c^d \sqrt{1+(f'(y))^2}\,dy$$

Ex. Find length of $x=y^2-4y$ from $(0,0)$ to $(-4,2)$

$$L=\int_0^2 \sqrt{1+(2y-4)^2}\,dy = 4.647$$

$y = \sqrt[3]{x}$ length from $(-27,-3)$ to $(1,1)$

$$L = \int_{-27}^{1} \sqrt{1 + \left(\frac{1}{3}x^{-2/3}\right)^2}\, dx$$

$\frac{1}{3x^{2/3}}$

Not diff @ $x=0$ (in interval) → Improper Integral

* To fix, need to do in terms of y!

$x = y^3$

$$L = \int_{-3}^{1} \sqrt{1 + (3y^2)^2}\, dy$$

Corners → Abs values → Piecewise → do in parts.

Ex. $y = x^2 - 4|x+1| - 6x$ $[-4,3]$ ($-(x+1)$)

$$y = \begin{cases} x^2 + 4(x+1) - 6x = x^2 - 2x + 4 & [-4,-1) \\ x^2 - 4(x+1) - 6x = x^2 - 10x - 4 & [-1,3] \end{cases}$$

$$\int_{-4}^{-1} \sqrt{1 + (2x-2)^2}\, dx + \int_{-1}^{3} \sqrt{1 + (2x-10)^2}\, dx$$

Ex. $y = x^3 + 2x|x-3| + 4 \qquad [-1,5]$

$$y = \begin{cases} x^3 - 2x(x-3) + 4 = x^3 - 2x^2 + 6x + 4 & [-1,3) \\ x^3 + 2x(x-3) + 4 = x^3 + 2x^2 - 6x + 4 & [3,5] \end{cases}$$

$$L = \int_{-1}^{3} \sqrt{1 + (3x^2 - 4x + 6)^2}\, dx + \int_{3}^{5} \sqrt{1 + (3x^2 + 4x - 6)^2}$$