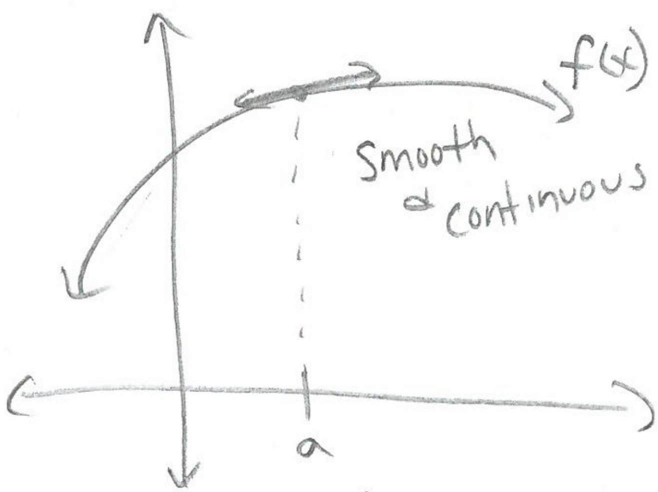
To be differentiable at $x=a$, derivative exists at $x=a$.

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\#$$

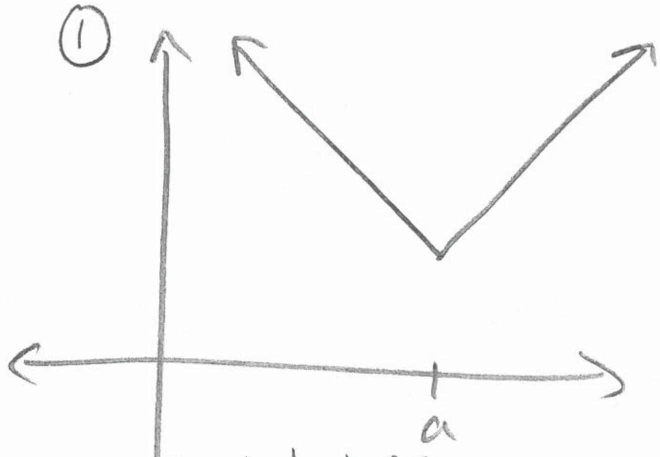
so $$\lim_{x\to a^-}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a^+}\frac{f(x)-f(a)}{x-a}$$

meaning one-sided derivatives are equal $\Rightarrow$ slopes from both directions as $x\to a$ are equal and not ∞.



* Diff @ $x=a$

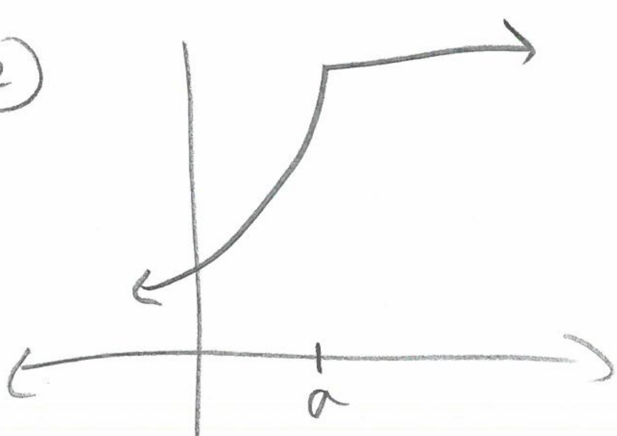
1



Not diff. @ $x=a$

Corner slopes = diff # from either side

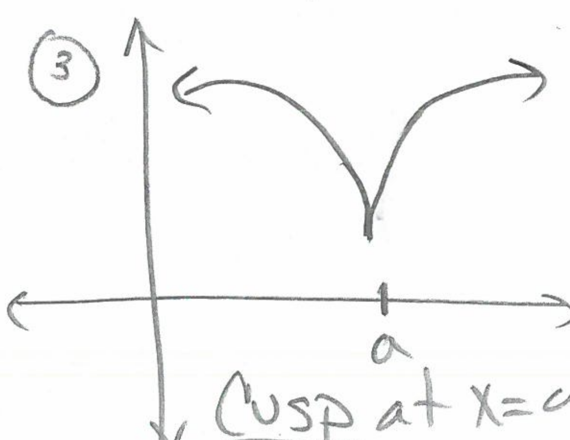
2



Not diff @ $x=a$

Corner

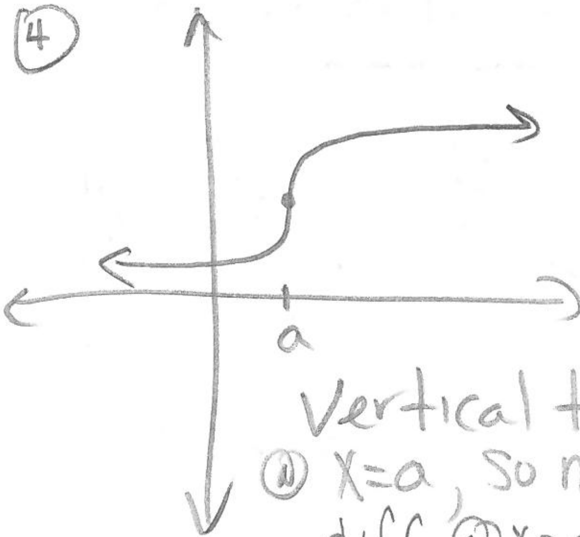
3



Cusp at $x=a$

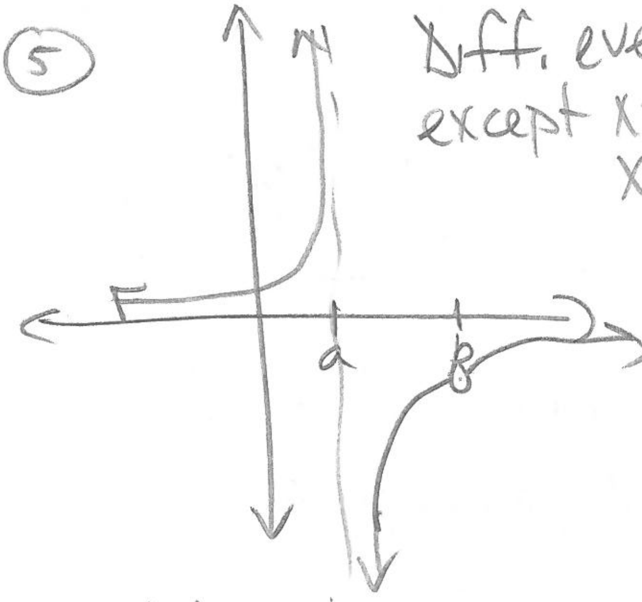
one sided deriv. go to different ∞. Not diff @ $x=a$

4



Vertical tangent @ $x=a$, so not diff. @ $x=a$.

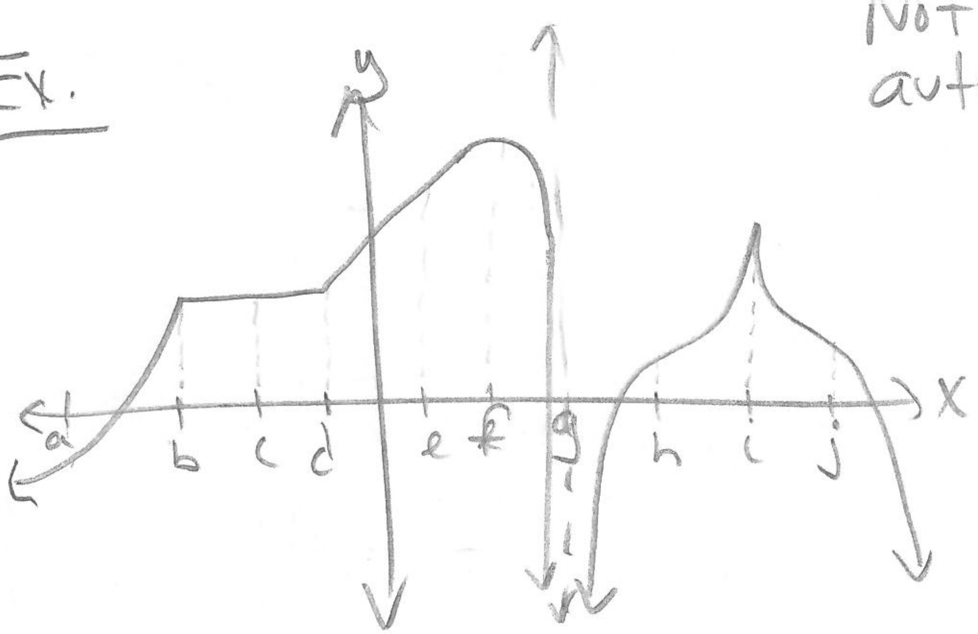
5



Diff. everywhere except $x=a$ & $x=b$.

Not cont @ $x=a$, automatically not diff. @ $x=a$.

Ex.



Not diff @ $x=$ b, d, g, i

Algebraically * Not diff where deriv. is not continuous.

$$y=|x|=\begin{cases}-x & x<0\\ x & x\geq 0\end{cases} \qquad y'=\begin{cases}-1 & x<0\\ 1 & x>0\end{cases}$$

B/c y' is not cont @ $x=0$ then y is not diff @ $x=0$.

$y=(x-2)^{1/3}$ Diff? $y'=\frac{1}{3}(x-2)^{-2/3}=\frac{1}{3(x-2)^{2/3}}$

domain $(-\infty,\infty)$
Cont.

y' DNE @ $x=2$ so
y is not diff. @ $x=2$.
<u>Is</u> diff $(-\infty,2)\cup(2,\infty)$

$y=x^{2/5}$ Diff? $y'=\frac{2}{5}x^{-3/5}=\frac{2}{5x^{3/5}}$

Cont. domain $(-\infty,\infty)$

y' DNE @ $x=0$
y not diff @ $x=0$.

$y=\sqrt{x}$ Where is diff?

domain: $[0,\infty)$
cont

$y'=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$\boxed{\text{Diff @ } (0,\infty)}$

$$y=\begin{cases} 2x^2-4x+1 & x<1 \\ x^3-2 & x\geq 1 \end{cases}$$

<u>Not</u> diff @ $x=1$

Is cont @ $x=1$? $\lim\limits_{x\to 1^-} 2x^2-4x+1=\lim\limits_{x\to 1^+} x^3-2=f(1)$

$-1\stackrel{?}{=}-1=f(1)$ ✓

Check deriv.

Is cont @ $x=1$

$$y'=\begin{cases} 4x-4 & x<1 \\ 3x^2 & x\geq 1 \end{cases}$$

$\lim\limits_{x\to 1^-} 4x-4 \stackrel{?}{\neq} \lim\limits_{x\to 1^+} 3x^2$
$0 \qquad 3$

y' not cont @ $x=1$
so y not diff.

Ex. Find the values of a & b so that $f(x)$ is differentiable

$$f(x) = \begin{cases} ax^2+bx+3 & x \le 1 \\ bx^3+x & x > 1 \end{cases}$$

① must be continuous!

$$\lim_{x \to 1^-} ax^2+bx+3 = \lim_{x \to 1^+} bx^3+x = f(1)$$

$$a+b+3 = b+1 \Rightarrow a = -2$$

② Deriv must be continuous!

$$f'(x) = \begin{cases} 2ax+b & x < 1 \\ 3bx^2+1 & x \ge 1 \end{cases}$$

$$\lim_{x \to 1^-} (2ax+b) = \lim_{x \to 1^+} (3bx^2+1)$$

$$2a+b = 3b+1$$

$$-4+b = 3b+1$$

$$-5 = 2b \Rightarrow \boxed{b = -5/2}$$