Worksheet: Inverse trigonometric functions
Mr. Chvatal **Name:**________________________ **Class:**______

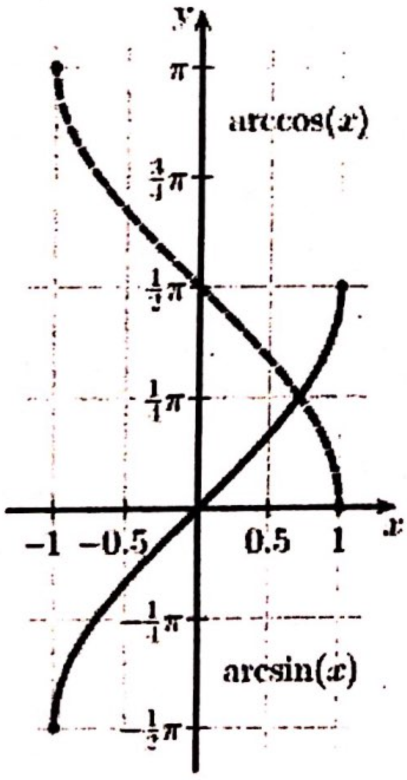
Introduction

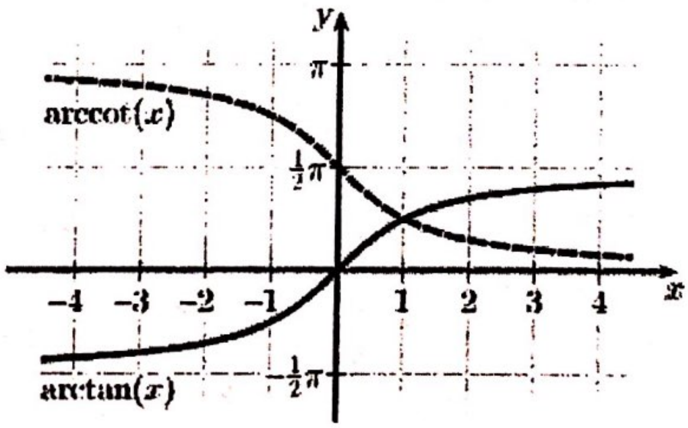
Inverse trigonometric functions are simply (or maybe not so simply) the inverses of the trig functions we just studied: sine, cosine and tangent.

There are two types of notation that describe an inverse trig function. The more common is what you'd find on a calculator keypad, and is consistent with the notation for other inverse functions we've studied: $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$. We can also write these same functions as arcsin *x*, arccos *x* and arctan *x*. We say (the first of) these as "the arcsine of *x*." You've used these inverses in geometry to find the measure of an unknown angle of a triangle. The arcsine of *x*, for example, is simply the angle whose sine is *x*. The arccosine of *x* is the angle whose cosine is *x*, etc.

It turns out none of these inverses are actually functions until we curtail them a bit. Picture the graph of sine or cosine wrapped around the *y*-axis instead of the *x*-axis, like a tree-climbing vine, as the inverse would appear to be. Clearly it won't pass the vertical line test. But if we restrict the range of arcsin *x*, arccos *x*, and arctan *x*, each can pass the vertical line test and meet the definition of a function.

Here are graphs of arcsin *x*, arccos *x* and arctan *x* (as well as arccot *x*; ignore this for now). Note that the axes are reversed from the trigonometric functions we just graphed. With inverse trig functions, $x = \sin y$, $x = \cos y$ and $x = \tan y$. We've exchanged *x* and *y*, just as we did with finding the inverse of linear functions.





✱ Determine:

$\lim_{x \to -\infty} \arctan x =$

$\lim_{x \to \infty} \arctan x =$

Function	Domain	Range
$y = \arcsin x$	$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$	$-\infty < x < \infty$	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

answers are in this range

FINISH

Use your unit circle to find the following:

1. $\arcsin\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$

2. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

3. $\sin^{-1} 2$ No solution

4. $\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

5. $\arccos(-1)$

6. $\arctan 0$

7. $\arctan(-1)$

8. $\arcsin\left(\frac{-\sqrt{2}}{2}\right)$

9. $\arccos\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

10. $\arccos\left(\frac{-\sqrt{2}}{2}\right)$

Do ODDS !

* KNOW your Unit circle !

10.6.5 EXERCISES

In Exercises 1 - 40, find the exact value.

1. $\arcsin(-1)$
2. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$
3. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$
4. $\arcsin\left(-\frac{1}{2}\right)$
5. $\arcsin(0)$
6. $\arcsin\left(\frac{1}{2}\right)$
7. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$
8. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$
9. $\arcsin(1)$
10. $\arccos(-1)$
11. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
12. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$
13. $\arccos\left(-\frac{1}{2}\right)$
14. $\arccos(0)$
15. $\arccos\left(\frac{1}{2}\right)$
16. $\arccos\left(\frac{\sqrt{2}}{2}\right)$
17. $\arccos\left(\frac{\sqrt{3}}{2}\right)$
18. $\arccos(1)$
19. $\arctan(-\sqrt{3})$
20. $\arctan(-1)$
21. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$
22. $\arctan(0)$
23. $\arctan\left(\frac{\sqrt{3}}{3}\right)$
24. $\arctan(1)$
25. $\arctan(\sqrt{3})$
26. $\operatorname{arccot}(-\sqrt{3})$
27. $\operatorname{arccot}(-1)$
28. $\operatorname{arccot}\left(-\frac{\sqrt{3}}{3}\right)$
29. $\operatorname{arccot}(0)$
30. $\operatorname{arccot}\left(\frac{\sqrt{3}}{3}\right)$
31. $\operatorname{arccot}(1)$
32. $\operatorname{arccot}(\sqrt{3})$
33. $\operatorname{arcsec}(2)$
34. $\operatorname{arccsc}(2)$
35. $\operatorname{arcsec}(\sqrt{2})$
36. $\operatorname{arccsc}(\sqrt{2})$
37. $\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right)$
38. $\operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right)$
39. $\operatorname{arcsec}(1)$
40. $\operatorname{arccsc}(1)$

In Exercises 41 - 48, assume that the range of arcsecant is $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ and that the range of arccosecant is $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$ when finding the exact value.

41. $\operatorname{arcsec}(-2)$
42. $\operatorname{arcsec}(-\sqrt{2})$
43. $\operatorname{arcsec}\left(-\frac{2\sqrt{3}}{3}\right)$
44. $\operatorname{arcsec}(-1)$
45. $\operatorname{arccsc}(-2)$
46. $\operatorname{arccsc}(-\sqrt{2})$
47. $\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)$
48. $\operatorname{arccsc}(-1)$

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Differentiation - Inverse Trigonometric Functions

Date________________ Period____

Differentiate each function with respect to x.

1) $y = \cos^{-1}(-5x^3)$

2) $y = \sin^{-1}(-2x^2)$

3) $y = \tan^{-1}(2x^4)$

4) $y = \csc^{-1}(4x^2)$

5) $y = (\sin^{-1} 5x^2)^3$

6) $y = \sin^{-1}(3x^5 + 1)^3$

7) $y = (\cos^{-1} 4x^2)^2$

8) $y = \cos^{-1}(-2x^3 - 3)^3$

1. $y = \sin^{-1}(2x)$

2. $y = \tan^{-1}(3x)$

3. $y = \sec^{-1}(e^{2x})$

4. $y = \sin^{-1}\sqrt{x}$

5. $y = \sin^{-1}\left(\frac{x}{3}\right)$

6. $y = \cos^{-1}(2x + 1)$

7. $y = \sec^{-1}(x^7)$

8. $y = \csc^{-1}(e^x)$

9. $y = \sin^{-1}\left(\frac{1}{x}\right)$

10. $y = e^x(\sec^{-1} x)$

11. $y = x^2(\sin^{-1} x)^3$