Particle Motion ⇒ Rectilinear Motion

* moving only up/down or only sideways (not both)

$s(t)$ ⇒ position in terms of time

↳ $x(t)$ ⇒ x position (moving sideways)

$y(t)$ ⇒ y position (moving up/down)

average velocity ⇒ $\frac{\text{change in position}}{\text{change in time}} = \frac{s(b)-s(a)}{b-a}$ on $[a,b]$

Units: ft/s, mph, m/s

instantaneous velocity ⇒ $s'(t) = v(t)$

$v(t)$ gives direction of motion

$v(t) > 0$ moving up/right

$v(t) < 0$ moving down/left

$v(t)$ ch. signs ⇒ changing direction

max position (max height) ⇒ $v(t)$ ch + to −

speed ⇒ $|v(t)|$ ⇒ magnitude of velocity

acceleration at time t: $a(t) = v'(t) = s''(t)$

Units: m/s^2, ft/s^2

* speed increasing or decreasing??

compare signs of $v(t)$ & $a(t)$

⇒ speed increasing if both $v(t)$ & $a(t)$ have same signs

⇒ speed decreasing if $v(t)$ & $a(t)$ have opp signs (working against each other)

$a(t) = 0$ ⇒ speed is constant

$s(t) = t^3 - 12t^2 + 36t - 20$ ft. $[-1, 9]$ sec. (sideway

a) Determine $v(t)$ and $a(t)$ at any time t.

$v(t) = s'(t) = 3t^2 - 24t + 36$ ft/s

$a(t) = v'(t) = 6t - 24$ ft/s^2

b) Determine $v(1)$ and $a(1)$ and describe the motion at that time (direction + speed incr/decr)

$v(1) = 3(1)^2 - 24(1) + 36 = 15$ ft/s $\quad a(1) = 6(1) - 24 = -18$

moving rt b/c $v(1) > 0$

speed decreasing b/c $v(1)$ & $a(1)$ have opp signs.

c) Describe the motion on $[-1, 9]$

+ do # lines with $v(t)$ & $a(t)$ + make chart with info.

$v(t) = 3t^2 - 24t + 36 = 3(t^2 - 8t + 12) = 3(t-6)(t-2) = 0$

$t = 6 \quad t = 2$

$a(t) = 6t - 24 = 6(t-4) = 0$

$t = 4$

v: + on $(-1, 2)$, − on $(2, 6)$, + on $(6, 9)$

a: − on $(-1, 4)$, + on $(4, 9)$

t	$v(t)$	direction	$a(t)$	speed incr/decr.
$(-1, 2)$	+	rt	−	decr
$t = 2$	0	changing	−	0
$(2, 4)$	−	left	−	incr
$t = 4$	−	left	0	constant
$(4, 6)$	−	left	+	decr
$t = 6$	0	changing	+	0
$(6, 9)$	+	rt	+	incr.

Particle Motion Practice

Given $s(t)$ describes the position of an object at time t. Determin the velocity and acceleration at any time t. Then find velocity and acceleration at time given. Describe direction and if speed is increasing or decreasing at that given time

1. $s(t) = 32t^2 + 3$; $t = 4$ 2. $s(t) = t + \sin t$; $t = \pi/2$

3. $s(t) = \sqrt[3]{t^2 + 2t + 1}$; $t = 0$ 4. $s(t) = \dfrac{\sin t}{t+1}$; $t = \pi$

Given $x(t)$ describes the horizontal position of an object at time t Describe the motion of the object on the interval (direction, spe for

5. $x(t) = t^2 - 2t + 2$; $[0, 5]$ 6. $x(t) = t^3 - 3t^2 + 4$; $[-2, 6]$

7. $x(t) = t^4 - 4t^3 + 4t^2 + 1$; $[-2, 6]$ 8. $x(t) = \dfrac{t}{t+1}$; $[-2, -1) \cup (-1$

9. $x(t) = \sin t$; $[0, 2\pi]$ 10. $x(t) = t^3 - 9t + 1$; $[-3, 3]$

11. A particle is moving such that it's vertical position at any time t is $y(t) = 2t^3 - 12t^2 + 6$ cm. At what time is the position changin the most rapidly?

12. A ball is thrown upward at 24 ft/sec from a 16 ft platform so that it's height at any time is $s(t) = 16 + 24t - 16t^2$ ft above the ground t seconds after thrown.
    a) Find the velocity and acceleration of the ball.
    b) When is the ball at its maximum height?
    c) How high will the ball go?
    d) When will the ball hit the ground?
    e) What is the velocity of the ball at impact?

Key Motion Practice

1. $v(t)=6t+4$ $a(t)=6$
$v(4)=256$ $a(4)=6$
moving up ($v(4)>0$) and speed increasing ($v(4)$ and $a(4)>0$)

2. $v(t)=1+\cos t$ $a(t)=-\sin t$
$v(\pi/2)=1$ $a(\pi/2)=-1$
moving up since $v(\pi/2)>0$ but speed decreasing since $v>0$ but $a(\pi/2)<0$.

3. $v(t)=\dfrac{2t+2}{3(t^2+2t+1)^{2/3}}$ $a(t)=$

$v(0)=2/3$ $a(0)=-\frac{2}{9}$

moving up, speed decreasing

4. $v(t)=\dfrac{\cos t(t+1)-\sin t}{(t+1)^2}$ $a(t)=\dfrac{-\sin t(t+1)^2-2(\cos t(t+1)-\sin t)}{(t+1)^3}$

$v(\pi)=\dfrac{-1}{\pi+1}$ $a(\pi)=\dfrac{2}{(\pi+1)^2}$

moving down since $v(\pi)<0$ and speed decr since $v(\pi)<0$ but $a(\pi)>0$.

5.

t	$v(t)$	dir	$a(t)$	Speed
$(0,1)$	$-$	left	$+$	$\downarrow$
$t=1$	0	ch	$+$	0
$(1,5)$	$+$	rt	$+$	$\uparrow$

6.

t	$v(t)$	dir	$a(t)$	speed
$(-2,0)$	$+$	rt	$-$	$\downarrow$
$t=0$	0	ch	$-$	0
$(0,1)$	$-$	left	$-$	$\uparrow$
$t=1$	$-$	left	0	constant
$(1,2)$	$-$	left	$+$	$\downarrow$
$t=2$	0	ch.	$+$	0
$(2,6)$	$+$	rt	$+$	$\uparrow$

7.

t	$v(t)$	dir	$a(t)$	spd
$(-2,0)$	$-$	left	$+$	$\downarrow$
$t=0$	0	ch	$+$	0
$(0,.4226)$	$+$	rt	$+$	$\uparrow$
$t=.4226$	$+$	rt	0	const
$(.4226,1)$	$+$	rt	$-$	$\downarrow$
$t=1$	0	ch	$-$	0
$(1,1.577)$	$-$	left	$-$	$\uparrow$
$t=1.577$	$-$	left	0	const
$(1.577,2)$	$-$	left	$+$	$\downarrow$
$t=2$	0	ch	$+$	0
$(2,6)$	$+$	rt	$+$	$\uparrow$

8.

t	$v(t)$	dir	$a(t)$	spd
$(-2,-1)$	$+$	rt	$+$	$\uparrow$
$(-1,2)$	$+$	rt	$-$	$\downarrow$

* $t=-1$ not in domain

9.

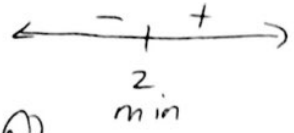
t	$v(t)$	dir	$a(t)$	spd
$(0,\pi/2)$	$+$	rt	$-$	$\downarrow$
$t=\pi/2$	0	ch	$-$	0
$(\frac{\pi}{2},\pi)$	$-$	left	$-$	$\uparrow$
$t=\pi$	$-$	left	0	const
$(\pi,3\pi/2)$	$-$	left	$+$	$\downarrow$
$t=3\pi/2$	0	ch	$+$	0
$(\frac{3\pi}{2},2\pi)$	$+$	rt	$+$	$\uparrow$

10.

t	$v(t)$	dir	$a(t)$	spd
$(-3,-\sqrt{3})$	$+$	rt	$-$	$\downarrow$
$t=-\sqrt{3}$	0	ch	$-$	0
$(-\sqrt{3},0)$	$-$	left	$-$	$\uparrow$
$t=0$	$-$	left	0	const.
$(0,\sqrt{3})$	$-$	left	$+$	$\downarrow$
$t=\sqrt{3}$	0	ch.	$+$	0
$(\sqrt{3},3)$	$+$	rt	$+$	$\uparrow$

11. $y(t) = 2t^3 - 12t^2 + 6$ * looking for extrema of $v(t)$

$v(t) = 6t^2 - 24t$

$a(t) = 12t - 24 = 0$ @ $t = 2$ 

"change most rapidly" can be max or min

Position change most rapid @ $t = 2$ sec. $v(2) = -24$ cm/sec.

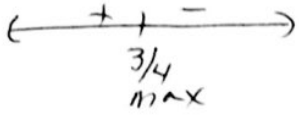
b/c $a(t)$ ch - to +

12. $s(t) = 16 + 24t - 16t^2$

a) $v(t) = s'(t) = 24 - 32t$ ft/s $\quad a(t) = v'(t) = -32$ ft/s^2

b) max height $\Rightarrow s'(t) = v(t) = 0 \quad 24 - 32t = 0$

"when" $\Rightarrow$ time t $\quad t = \frac{24}{32} = \frac{3}{4}$ sec b/c $v(t)$ ch + to -

c) How high? Find $s(3/4) = 25$ ft. 

d) When hit ground $\Rightarrow s(t) = 0$

$16 + 24t - 16t^2 = 0 \Rightarrow t = 2$ sec.

e) Plug ans. from (d) into $v(t) \Rightarrow v(2) = 24 - 32(2) = -40$ ft/sec.