AP Calculus BC: Parametric/Vector Motion

Key

1. A particle is moving such that its position (in cm) at time t seconds is $\langle t^3 - 3t^2 + 4t, \sin t \rangle$.

   a) Find $\vec{v}(t)$ and $\vec{a}(t)$. $\vec{v}(t) = \langle 3t^2 - 6t + 4, \cos t \rangle$ $\vec{a}(t) = \langle 6t - 6, -\sin t \rangle$

   b) Describe the motion of the particle at t = 1 second.
   $\vec{v}(1) = \langle 1, .5403 \rangle$ $\vec{a}(1) = \langle 0, -.8415 \rangle$
   moving rt at constant speed since $x'(1) > 0$ & $x''(1) = 0$
   moving up & speed dec. since $y'(1) > 0$ and $y''(1) < 0$.

   c) Find the speed at 2 seconds. 4.0216 cm/s

   d) Find the total distance traveled on [0, 4].
   32.271 cm

2. An object moves along a curve in the xy-plane with position $\langle x(t), y(t) \rangle$ at time t seconds for $0 \le t \le 5$ and $\frac{dx}{dt} = \sin(t^2)$ and $\frac{dy}{dt} = t^3 \cos t$. At t = 1, the object is at position $\langle -2, 4 \rangle$.

   a) Find the speed at t = 1 second. 1

   b) Find the total distance traveled on [0, 5].
   $$76.720 = \int_0^5 \sqrt{(\sin(t^2))^2 + (t^3 \cos t)^2}\, dt$$

   c) Find the displacement on [0, 5].
   $\langle .5279, -65.525 \rangle$

   d) Find the position at t = 4.
   $(-1.563, -47.897)$

   $\int_1^4 \sin(t^2)\, dt = x(4) + 2$
   $\int_1^4 t^3 \cos t\, dt = y(4) - 4$

3. The acceleration of an object moving in the xy-plane is given by $\left\langle t - 1, \frac{1}{t} \right\rangle$ for the time interval [1, 6]. At t = 1, the object is at (2, 1) and at rest.

   a) Find $\vec{v}(t)$ and the position vector $\langle x(t), y(t) \rangle$.
   $\vec{v}(t) = \left\langle \frac{1}{2}t^2 - t + \frac{1}{2}, \ln|t| \right\rangle$
   $\vec{r}(t) = \left\langle \frac{1}{6}t^3 - \frac{1}{2}t^2 + \frac{1}{2}t + \frac{11}{6}, t\ln|t| - t + 2 \right\rangle$

   b) When is the object farthest to the left? What is its position at this time? Justify your answer.
   $x'(t) = 0 = \frac{1}{2}(t^2 - 2t + 1) = \frac{1}{2}(t-1)^2 = 0$ ck $t = 1$ & $t = 6$
   crit #: $t = 1$ @1, $x = 2$ @6, $x = 22.8\overline{3}$
   Farthest left @ $t = 1$ position $(2, 1)$

   c) The object starts from rest at t = 1. Is there any other time that the object is stopped on [1, 6]? If so, when? Explain why or why not. No, $\frac{dx}{dt}$ & $\frac{dy}{dt}$ never $= 0$ simultaneously

   d) Determine the speed at t = 3 seconds.
   2.282