arametric + Polar Review

1. $\frac{dy}{dx} = \boxed{\frac{-1}{2t^3}}\Big|_{t=2} = -\frac{1}{16} \Rightarrow \boxed{y - 3/2 = -\frac{1}{16}(x-3)}$

$\frac{d^2y}{dx^2} = \frac{\frac{3}{2}t^{-4}}{2t} = \boxed{\frac{3}{4t^5}}\Big|_{t=2} \Rightarrow \frac{3}{128} > 0$ so f is concave up.

2. $\frac{dy}{dx} = \frac{2\sec^2 t}{\sec t \tan t} = 2\csc t\Big|_{t=\frac{\pi}{6}} = 4$

Sorry! Same as #17 on other review!

Aarrgh ⇒ Same as #18 on other review!!

4. $\frac{dy}{dx} = \frac{8t}{\frac{1}{2\sqrt{t}}} = \boxed{16t^{3/2}}$ @ $t=4 = 128$ $\boxed{y-63=128(x-2)}$

$\frac{d^2y}{dx^2} = \frac{24t^{1/2}}{\frac{1}{2\sqrt{t}}} = \boxed{48t}$ @ $t=4 \Rightarrow 192 > 0$ so c. up.

ther 4 (oops!)

1. horiz: $\frac{dy}{dt} = 3t^2 - 3 = 0$, $t = \pm 1$ $\boxed{(2,-2) \;\&\; (4,2)}$

vert: $\frac{dx}{dt} = 2t - 1 = 0$, $t = \frac{1}{2}$ $\boxed{(1.75, -11/8)}$

$x = (1-\cos\theta)\cos\theta$

$y = (1-\cos\theta)\sin\theta$

oriz: $\frac{dy}{d\theta} = \sin^2\theta + (1-\cos\theta)\cos\theta$
$= \sin^2\theta - \cos^2\theta + \cos\theta$
$= -2\cos^2\theta + \cos\theta + 1 = 0$
$= -(2\cos\theta + 1)(\cos\theta - 1) = 0$
$\cos\theta = -\frac{1}{2}$ $\cos\theta = 1$
$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\theta = \cancel{0}$ cusp @ $\theta = 0$

Plug into x+y

$\boxed{\left(-\frac{3}{4}, \frac{3\sqrt{3}}{4}\right) \left(-\frac{3}{4}, \frac{-3\sqrt{3}}{4}\right)}$

Vert: $\frac{dx}{d\theta} = \sin\theta\cos\theta - (1-\cos\theta)\sin\theta$
$= 2\sin\theta\cos\theta - \sin\theta = 0$
$= \sin\theta(2\cos\theta - 1) = 0$
$\theta = \cancel{0}, \pi$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$
$\boxed{(-2,0) \left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right) \left(\frac{1}{4}, \frac{-\sqrt{3}}{4}\right)}$

$x = 3\sin\theta\cos\theta$ $[0, \pi)$

$y = 3\sin^2\theta$

oriz: $\frac{dy}{d\theta} = 6\sin\theta\cos\theta = 0$
$\theta = 0, \pi/2$ $\boxed{(0,0)\,(0,3)}$

rt: $\frac{dx}{d\theta} = 3\cos^2\theta - 3\sin^2\theta = 0$
$\theta = \pi/4, 3\pi/4$ $\boxed{(3/2, 3/2) \left(-\frac{3}{2}, \frac{3}{2}\right)}$

7. $\int_{-1}^{1} \sqrt{(2t)^2 + (12t^2)^2}\, dt$

8. $\int_0^{\pi/2} \sqrt{(-e^{-t}\cos t - e^{-t}\sin t)^2 + (-e^{-t}\sin t + e^{-t}\cos t)^2}$

10. a) $\vec{v}(t) = \langle t+2, -e^{-t} + 2 \rangle$
position $= \langle \frac{1}{2}t^2 + 2t + 1, e^{-t} + 2t \rangle$

b) @ $t = 2$ $(7, e^{-2} + 4)$

9. a) $\vec{v}(t) = \langle -4\cos t, 4(1-\sin t) \rangle$ $\vec{a}(t) = \langle 4\sin t, -4\cos t \rangle$

b) $\sqrt{(2)^2 + (4-2\sqrt{3})^2} = \boxed{2.071 \text{ ft/s}^2}$

c) $\int_1^3 \sqrt{(-4\cos t)^2 + (4-4\sin t)^2}\, dt = \boxed{4.562 \text{ ft}}$

11. $\frac{1}{2}\int_{-\pi/10}^{\pi/10} (3\cos 5\theta)^2\, d\theta = \boxed{1.4}$

12. $2\left[\frac{1}{2}\int_{\sin^{-1}(\frac{1}{4})}^{\pi/2} (3\sin\theta)^2\, d\theta - \frac{1}{2}\int_{\sin^{-1}(\frac{1}{4})}^{\pi/2} (1-\sin\theta)^2 d\right.$ $= \boxed{6.859}$

13. $\frac{1}{2}\int_0^{2\pi} (2-4\cos\theta)^2\, d\theta - 2 \cdot \frac{1}{2}\int_{-\pi/3}^{\pi/3} (2-4\cos\theta)^2\, d\theta$

2012 SA #2

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Because $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055$ (or 3.054)

3 : { 1 : moving to the right with reason; 1 : considers $\frac{dy/dt}{dx/dt}$; 1 : slope at $t = 2$ }

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\, dt = 1.253$ (or 1.252)

2 : { 1 : integral; 1 : answer }

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

2 : { 1 : speed; 1 : acceleration }

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$
$= 0.651$ (or 0.650)

2 : { 1 : integral; 1 : answer }