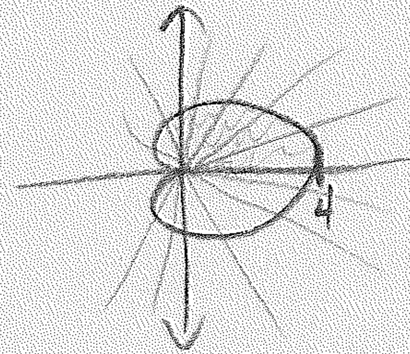
Polar Area



Sectors

$A_{sector} = \frac{1}{2} r^2 \theta$

$A = \int \frac{1}{2} r^2 \, d\theta$

$= \frac{1}{2} \int_{\alpha}^{\beta} r^2 \, d\theta$ $\Rightarrow$ On interval $[\alpha, \beta]$ r value is always postive or always neg. (not mixed)

Find area bounded by $r = 2 + 2\cos\theta$ $[0, 2\pi)$

$$A = \frac{1}{2} \int_0^{2\pi} (2 + 2\cos\theta)^2 \, d\theta \approx 18.84956 \approx 18.850$$

Find area of $r = 4\sin\theta$. $[0, \pi)$

$$\frac{1}{2} \int_0^{\pi} (4\sin\theta)^2 \, d\theta$$

Find area of one petal of rose curve given by $r = 3\cos 3\theta$. To get interval, find When @ pole?

$$\frac{1}{2} \int_{\pi/6}^{\pi/2} (3\cos 3\theta)^2 \, d\theta \text{ ?}$$

$3\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

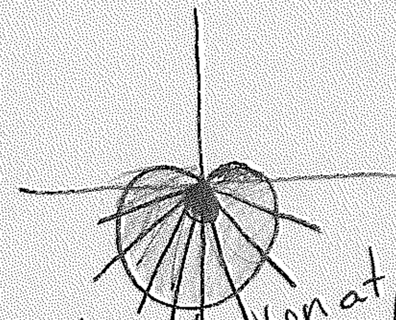
$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \ldots$

$$\frac{1}{2} \int_{\pi/2}^{5\pi/6} (3\cos 3\theta)^2 \, d\theta$$

$$\int_0^{\pi/6} (3\cos 3\theta)^2 \, d\theta$$

Use closest interval for one petal

$r = 1 - 2\sin\theta$



a) Find area of inner loop.

$$\frac{1}{2}\int_{\pi/6}^{5\pi/6} (1-2\sin\theta)^2 \, d\theta$$

To get interval, find when at pole

$1 - 2\sin\theta = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

b) Find area between inner + outer loops.

$$\frac{1}{2}\int_{0}^{\pi/6} (1-2\sin\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{5\pi}{6}}^{2\pi} (1-2\sin\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1-2\sin\theta)^2 \, d\theta$$

OR

$$\frac{1}{2}\int_{0}^{2\pi} (1-2\sin\theta)^2 \, d\theta - 2\cdot\frac{1}{2}\int_{\pi/6}^{5\pi/6} (1-2\sin\theta)^2 \, d\theta$$