Tangent Lines in Polar

slope of tangent line $= \frac{dy}{dx}$

* turn polar into parametric for tangents

$x = r\cos\theta$ $\qquad$ $r = f(\theta)$

$y = r\sin\theta$ $\qquad$ $x = f(\theta)\cos\theta$

$y = f(\theta)\sin\theta$

Slope tangent $= \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

horiz tangents $\Rightarrow \frac{dy}{d\theta} = 0$

vert tangents $\Rightarrow \frac{dx}{d\theta} = 0$

if both $= 0$ at same θ, then not horiz or vert
$\Rightarrow$ cusp

Ex. $r = 2 - 2\cos\theta$

① Find eqn. of tangent at $\theta = \pi/2$

$x = (2-2\cos\theta)\cos\theta$
$y = (2-2\cos\theta)\sin\theta$ > Plug in $\frac{\pi}{2}$ to get pt. $(0, 2)$

$$\frac{dy}{dx} = \frac{2\sin\theta\sin\theta + (2-2\cos\theta)\cos\theta}{2\sin\theta\cos\theta - (2-2\cos\theta)\sin\theta}\Bigg|_{\theta=\frac{\pi}{2}} = \frac{2}{-2} = -1$$

$y - 2 = -1(x-0) \Rightarrow \boxed{y = -x+2}$

Horiz & vert tangents ⇒ give polar pts

Horiz: $\frac{dy}{d\theta} = 0 = 2\sin^2\theta + 2\cos\theta - 2\cos^2\theta$

$= 2(1-\cos^2\theta) + 2\cos\theta - 2\cos^2\theta = 0$

$= -4\cos^2\theta + 2\cos\theta + 2 = -2(2\cos^2\theta - \cos\theta - 1) = 0$

$-2(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ $\cos\theta = 1$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ ~~$\theta = 0$~~

polar pts $[r, \theta]$

$\left[3, \frac{2\pi}{3}\right]$

$\left[3, \frac{4\pi}{3}\right]$

Vert: $\frac{dx}{d\theta} = 0 = 2\sin\theta\cos\theta - 2\sin\theta + 2\sin\theta\cos\theta$

$= 4\sin\theta\cos\theta - 2\sin\theta = 2\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ $\cos\theta = \frac{1}{2}$

~~$\theta = 0, \pi$~~ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

polar pts

$[4, \pi]$ $\left[1, \frac{\pi}{3}\right]$

$\left[1, \frac{5\pi}{3}\right]$

Tangents at pole ⇒ tangents that pass through pole (origin)

only ones that don't use x+y.

Find where $r=0$ and $\frac{dr}{d\theta} \neq 0$

Ans ⇒ $\theta = \#$ → convert to rect. if needed.

Ex. $r = 2\sin 3\theta$. Find tangents at pole.

Rose w/ 3 petals
int: $[0, \pi)$

$r=0 \rightarrow 2\sin 3\theta = 0$

$\sin 3\theta = 0$

$3\theta = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\frac{dr}{d\theta} = 6\cos 3\theta \neq 0$ at

Polar

$\theta = 0$

$\theta = \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

Rect. → Convert

$\tan\theta = \tan 0$

$\frac{y}{x} = 0 \Rightarrow$ $y=0$

$\tan\theta = \tan\frac{\pi}{3}$

$\frac{y}{x} = \sqrt{3} \Rightarrow$ $y = \sqrt{3}x$

$\tan\theta = \tan\frac{2\pi}{3}$

$\frac{y}{x} = -\sqrt{3}$

$y = -\sqrt{3}x$