p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p \le 1$ series div.
$p > 1$ series conv.

Comparison Tests $\Rightarrow$ only for pos. termed series

① Direct or Basic Comparison

$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are pos. termed series

a_n: want to know

b_n: known series

If $\sum_{n=1}^{\infty} b_n$ converges and $a_n \le b_n$, then $\sum_{n=1}^{\infty} a_n$ conv. too.

If $\sum_{n=1}^{\infty} b_n$ diverges and $a_n \ge b_n$, then $\sum_{n=1}^{\infty} a_n$ div. too

Ex. $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$ (a_n) $b_n = \frac{1}{5^n} = \left(\frac{1}{5}\right)^n$ Geo $r = \frac{1}{5} < 1$ so Conv.

$$\frac{1}{2+5^n} \le \frac{1}{5^n} \text{ for } n \ge 1$$

$\therefore$ By basic comp. since $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$ conv.

$\sum_{n=1}^{\infty} \frac{1}{2+5^n}$ also conv.

Ex. $\sum_{n=2}^{\infty} \frac{3}{\sqrt{n}-1} = a_n$ $b_n = \frac{3}{\sqrt{n}} = 3 \cdot \frac{1}{\sqrt{n}}$ p series $p = \frac{1}{2} < 1$ so diverges

$$\frac{3}{\sqrt{n}-1} \ge \frac{3}{\sqrt{n}} \text{ for } n \ge 2$$

$\therefore$ By basic comp. since $\sum_{n=2}^{\infty} \frac{3}{\sqrt{n}}$ div., $\sum_{n=2}^{\infty} \frac{3}{\sqrt{n}-1}$ diverges too

② Limit Comparison Test

$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are pos. termed series
$\rightsquigarrow$ known

$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$. If L is a # > 0, then either both series conv. or both series diverge.

* Most used when a_n is a rational function but not p series

Ex. $\sum_{n=1}^{\infty} \frac{n+1}{3n^3-2n+7} = a_n$ $\qquad b_n = \frac{n}{n^3} = \frac{1}{n^2}$

p series $p = 2 > 1$ so converges.

$$\lim_{n\to\infty}\left(\frac{n+1}{3n^3-2n+7} \div \frac{1}{n^2}\right) = \lim_{n\to\infty}\left(\frac{n+1}{3n^3-2n+7}\cdot\frac{n^2}{1}\right)$$
$$= \frac{1}{3} > 0$$

$\therefore$ By limit comp., since $\sum_{n=1}^{\infty} b_n$ converges, $\sum_{n=1}^{\infty} a_n$ conv. too

Ex. $\sum_{n=1}^{\infty} \frac{n^2-2}{3^n(2n^2-3n+4)} = a_n$ $\qquad b_n = \frac{\cancel{n^2}}{3^n \cancel{n^2}} = \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$

geo $r = \frac{1}{3} < 1$ so conv.

$$\lim_{n\to\infty}\left[\frac{n^2-2}{\cancel{3^n}(2n^2-3n+4)}\cdot\frac{\cancel{3^n}}{1}\right] = \frac{1}{2} > 0$$

$\therefore$ By lim comp., since $\sum_{n=1}^{\infty} b_n$ conv., $\sum_{n=1}^{\infty} a_n$ conv too

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}} = a_n$$

$b_n = \frac{1}{\sqrt{n}}$ p series $p = \frac{1}{2} < 1$ so diverges

OR $b_n = \frac{1}{\sqrt{3n}} = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{n}}$

$$\lim_{n \to \infty} \left(\frac{1}{\sqrt{3n-2}} \cdot \frac{\sqrt{n}}{1} \right) = \frac{1}{\sqrt{3}} > 0$$

$$\frac{1}{\sqrt{3n-2}} \geq \frac{1}{\sqrt{3n}}$$

∴ By limit comp since $\sum_{n=1}^{\infty} b_n$ div., $\sum_{n=1}^{\infty} a_n$ div. also.

p. 587 # 3, 5, 8, 12, 15-23 odd, 24, 29-36 all