Cross-sectional Volumes

Already solid - not rotating

Typically base in xy plane ⇒ region bounded by curves

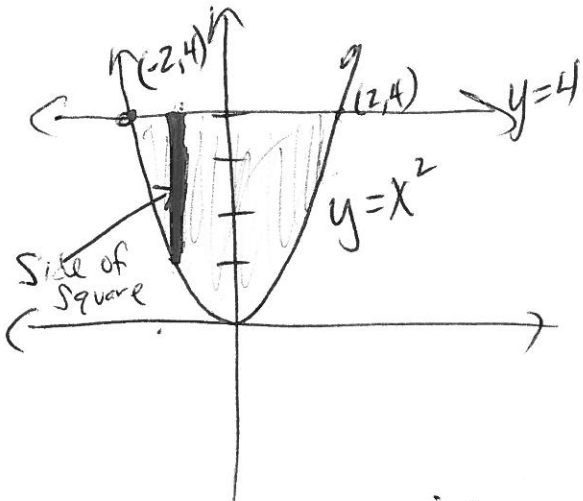
cut into solid ⊥ to x axis : terms of x

⊥ to y axis: terms of y

In general, Volume = (Area base of cross-sect.)(height)

$$\text{Volume} = \int_a^b (\text{Area of cross-section})\, dx \text{ or } dy$$

Ex. A solid has base in xy-plane bounded by $y=4$ and $y=x^2$. Find the volume of solid if cross-sections are...



a) ⊥ to x-axis; squares

(x)

$A = s^2$

$4 - x^2 = s$

$A = (4-x^2)^2$

$$V = \int_{-2}^{2} (4-x^2)^2\, dx$$

b) ⊥ x-axis; semicircles

(x)

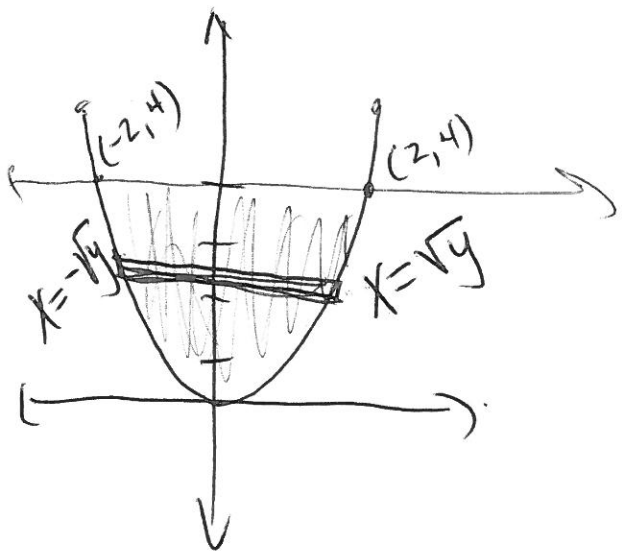
$A = \frac{1}{2}\pi r^2$

$4 - x^2 =$ diameter

radius $= \frac{4-x^2}{2}$

$$\int_{-2}^{2} \frac{1}{2}\pi \left(\frac{4-x^2}{2}\right)^2 dx$$

⊥ to y-axis; isosceles rt triangles with leg in xy-plane $A = \frac{1}{2}bh$



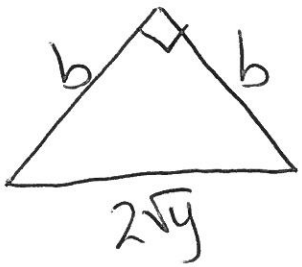
(y) $A = \frac{1}{2}b^2$

$\sqrt{y} - -\sqrt{y} = 2\sqrt{y} = b$

$$\int_0^4 \frac{1}{2}(2\sqrt{y})^2 \, dy$$

⊥ to y-axis; isosceles rt △ with hypotenuse in xy plane

(y) $A = \frac{1}{2}b^2$



$b^2 + b^2 = (2\sqrt{y})^2$

$2b^2 = (2\sqrt{y})^2$

$b^2 = \frac{(2\sqrt{y})^2}{2}$

$$\int_0^4 \frac{1}{2}\frac{(2\sqrt{y})^2}{2} \, dy$$