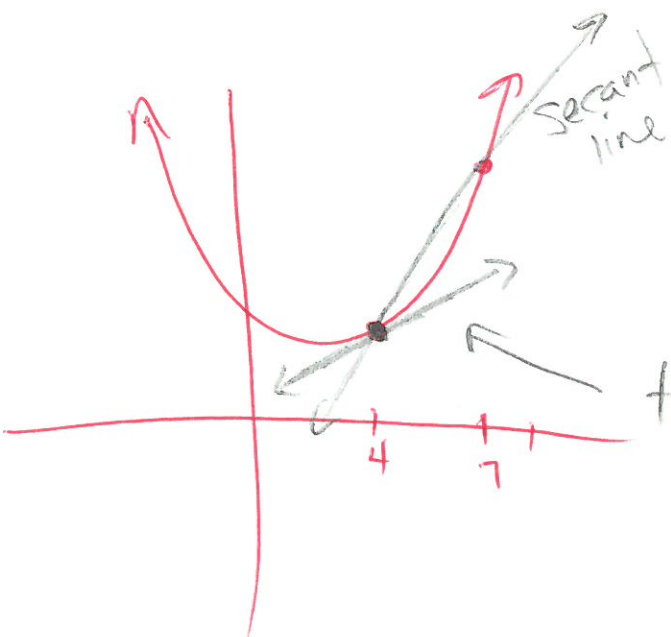
Average rate of change = Slope

The population (in thous.) of a town t years after 1990 can be found by $P(t) = 3t^2 - t + 1$. Find the average of change of the population from 1994 to 1997.

$$\frac{\Delta \text{ population}}{\Delta \text{ time}} = \frac{P(7)-P(4)}{7-4} = \frac{141-4}{3}$$

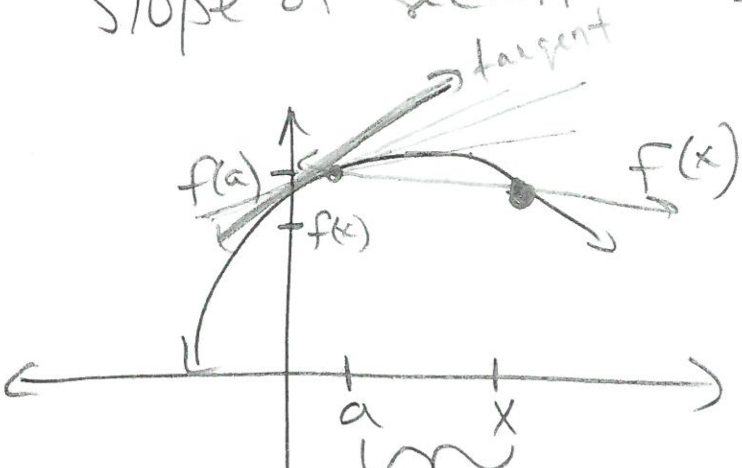
$= 32$ thous people/yr.



Slope of secant = average rate of change

Slope tangent line = instantaneous rate of change

$$\text{slope of secant} = \frac{f(x)-f(a)}{x-a} = \frac{f(a+\Delta x)-f(a)}{\Delta x}$$



$\Delta x = x - a$

$h = \Delta x$

Slope of tangent at $x=a$ $\Rightarrow \lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$

$\Rightarrow \lim\limits_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$

$\Rightarrow \lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ = inst. rate of $f(x)$ at any x
= slope of tangent at any x
= slope of curve at any x
= <u>derivative</u> of $f(x)$

$\Rightarrow f'(x) \Rightarrow y'$

$\frac{d}{dx}[f(x)] \Rightarrow \frac{d}{dx}[y] \Rightarrow \frac{dy}{dx}$

↑
indep. var. is x

$f(x) = 2x^2 - x + 6$ Find $f'(x)$.

$$\lim_{h\to 0} \frac{2(x+h)^2 - (x+h) + 6 - (2x^2 - x + 6)}{h}$$

p. 102 #5-23 odd

$f(x) = 2x^2 - 3x + 4$. Find the slope of tangent at $x = 2$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) + 4 - (2x^2 - 3x + 4)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2 + 2hx + h^2) - 3x - 3h + 4 - 2x^2 + 3x - 4}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 \cancel{- 3x} - 3h \cancel{+4} \cancel{- 2x^2} \cancel{+ 3x} \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = 4x - 3 = f'(x)$$

$$f'(2) = 4(2) - 3 = 5$$

$$f'(-10) = -43$$

Find equation of tangent to $f(x)$ at $x = 2$.

$f(x) = 2x^2 - 3x + 4$

deriv slope $= f'(2) = 5$

point $= (2, 6)$ y value $\Rightarrow$ plug 2 into orig. $f(x)$

pt-slope form: $y - y_1 = m(x - x_1)$

$$\boxed{y - 6 = 5(x - 2)}$$