

Separable Differential Equations

Find the general solution of each differential equation.

1) $\frac{dy}{dx} = e^{x-y}$

$$e^y = e^x + C$$

$$y = \ln(e^x + C)$$

2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$$\tan y = x + C$$

$$y = \tan^{-1}(x + C)$$

$y(2) = 0$

$\tan 0 = 2 + C$

$C = -2$

$$y = \tan^{-1}(x - 2)$$

3) $\frac{dy}{dx} = xe^y$

$$-e^{-y} = \frac{x^2}{2} + C_1$$

$$y = -\ln\left(-\frac{x^2}{2} + C\right)$$

4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$; $y(1) = 0$

$$\frac{e^{2y}}{2} = x^2 + C_1$$

$$y = \frac{\ln(2x^2 + C)}{2}$$

$$y = \frac{\ln(2x^2 - 1)}{2}$$

$\Rightarrow 0 = \frac{\ln(2 + C)}{2}$

$e^0 = \ln(2 + C)$

$1 = 2 + C$

$C = -1$

5) $\frac{dy}{dx} = 2y - 1$; $y(0) = 4$

$$\frac{\ln|2y - 1|}{2} = x + C_1$$

$$y = \frac{Ce^{2x} + 1}{2}$$

$$4 = \frac{Ce^0 + 1}{2}$$

$$8 = C + 1$$

$$C = 7$$

$$y = \frac{7e^{2x} + 1}{2} = \frac{7}{2}e^{2x} + \frac{1}{2}$$

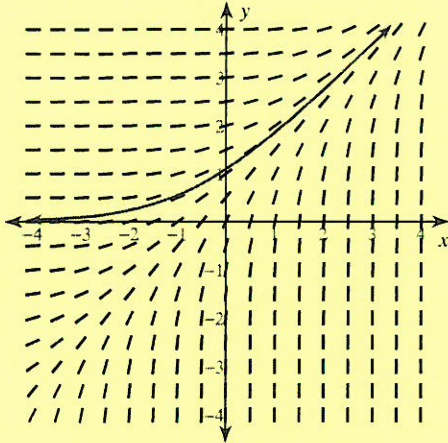
6) $\frac{dy}{dx} = 2yx + yx^2$

$$\ln|y| = x^2 + \frac{x^3}{3} + C_1$$

$$y = Ce^{x^2 + \frac{x^3}{3}}$$

For each problem, find the particular solution of the differential equation that satisfies the initial condition. You may use a graphing calculator to sketch the solution on the provided graph.

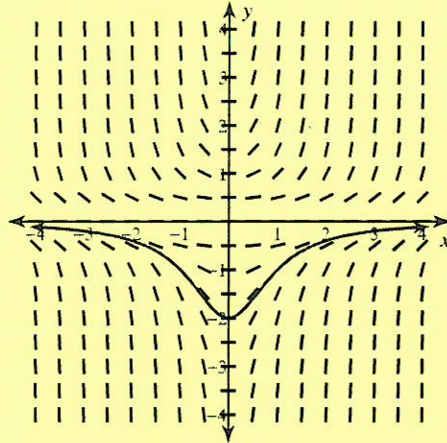
7) $\frac{dy}{dx} = 2e^{x-y}, y(1) = \ln(2e + 1)$



$$e^y = 2e^x + 1$$

$$y = \ln(2e^x + 1)$$

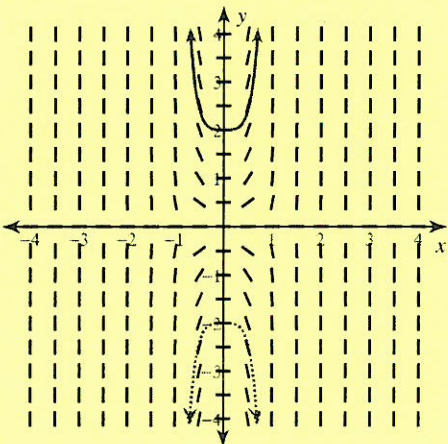
8) $\frac{dy}{dx} = xy^2, y(2) = -\frac{2}{5}$



$$-\frac{1}{y} = \frac{x^2}{2} + \frac{1}{2}$$

$$y = -\frac{2}{x^2 + 1}$$

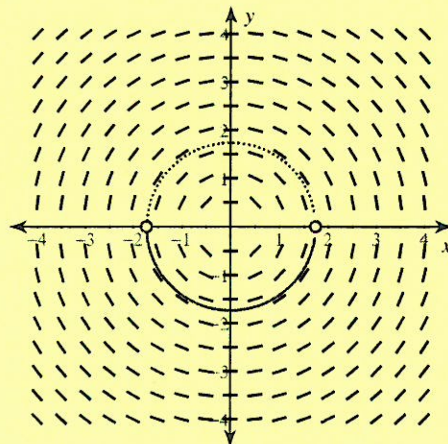
9) $\frac{dy}{dx} = 12x^3y, y(0) = 2$



$$\ln|y| = 3x^4 + \ln 2$$

$$y = 2e^{3x^4}$$

10) $\frac{dy}{dx} = -\frac{x}{y}, y(1) = -\sqrt{2}$



$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2} \rightarrow y = \pm\sqrt{-x^2 + 3}$$

$$y = -\sqrt{-x^2 + 3}, \sqrt{3} < x < \sqrt{3}$$

Choose - from \pm b/c of condition $y(1) = -\sqrt{2}$.