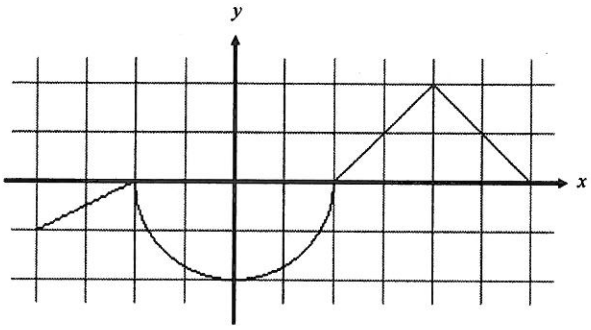
KEY

AP Calculus
5.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The graph of f shown below consists of line segments and a semicircle. Evaluate each definite integral.



a) $\int_0^2 f(x)dx$

$-\pi$

b) $\int_2^6 f(x)dx$

4

c) $\int_{-4}^2 f(x)dx$

$-1-2\pi$

d) $\int_{-4}^6 f(x)dx$

$3-2\pi$

e) $\int_{-4}^6 |f(x)|dx$

$5+2\pi$

f) $\int_{-4}^6 [f(x)+2]dx$

$= \int_{-4}^6 f(x)dx + \int_{-4}^6 2\,dx$

$= 3-2\pi + 2(10) = 23-2\pi$

2. Part *e* above, gives a way to find the total <u>area</u> between the x – axis and the function between $x = -4$ and $x = 6$. *Without using absolute value signs*, write two different expressions that can be used to find the total <u>area</u> between the x–axis and the function between $x = -4$ and $x = 6$.

3. Suppose that f and g are continuous and $\int_1^2 f(x)\,dx = -4$, $\int_1^5 f(x)dx = 6$, and $\int_1^5 g(x)dx = 8$.

Find each of the following:

a) $\int_2^2 g(x)dx$

0

b) $\int_5^1 g(x)dx$

-8

c) $\int_1^2 3f(x)\,dx$

-12

d) $\int_2^5 f(x)dx$

10

e) $\int_1^5 [f(x)-g(x)]dx$

-2

f) $\int_1^5 [f(x)+4]dx$

22

$= \int_1^5 f(x)dx + \int_1^5 4\,dx$

$= 6 + 4(4) = 22$

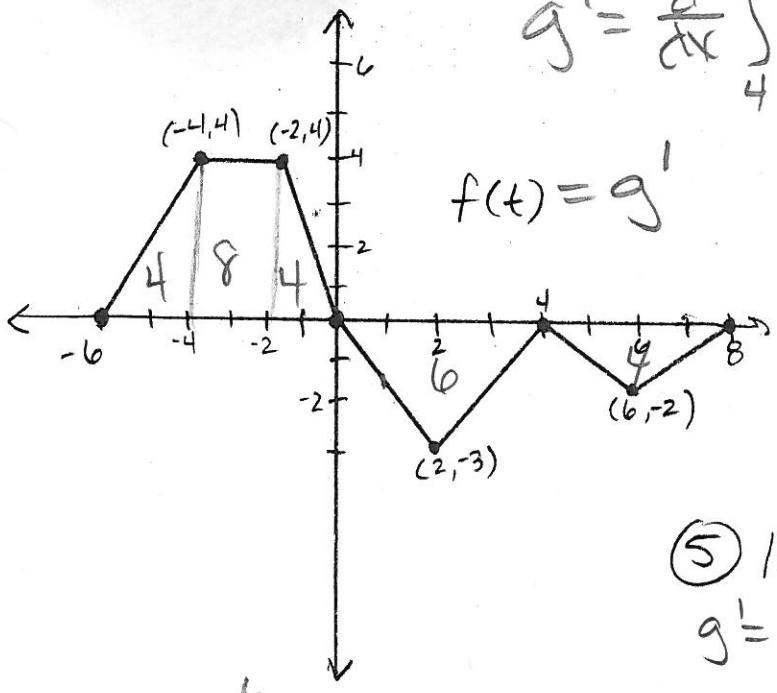
$\uparrow$

$(5-1)$

KEY

$g' = \frac{d}{dx}\int_4^x f(t)\,dt = f(x)$

Given $g(x) = \int_4^x f(t)\,dt$ on $[-6,8]$



Determine the following:

① $g(0) = 6$ ② $g(8) = -4$

③ $g'(2) = -3$ ④ $g''(1) = -3/2$

⑤ Intervals where $g(x)$ increasing?

$g' = f > 0$ $(-6,0)$

$g(x)$ decreasing?

$g' = f < 0$ $(0,4) \cup (4,8)$

$g(x)$ concave up? $g' = f$ incr. $(-6,-4) \cup (2,4) \cup (6,8)$

$g' = f$ dec. $(-2,0) \cup (0,2) \cup (4,6)$ down?

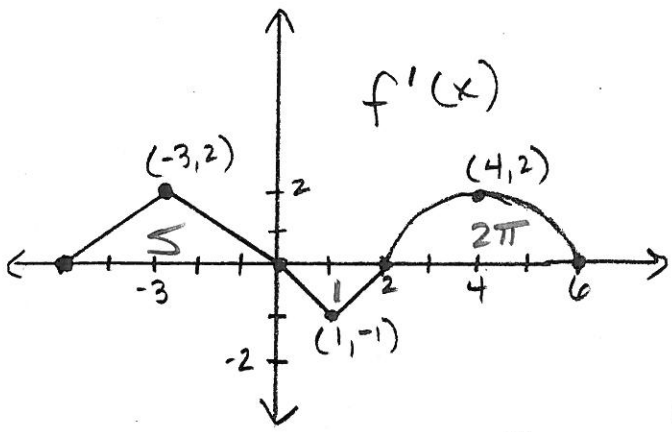
⑥ Abs max value on $[-6,8]$ of $g(x)$? 6

Abs min value on $[-6,8]$ of $g(x)$? -10

$g(-6) = \int_4^{-6} f(t)\,dt = -10$

$g(8) = \int_4^8 f(t)\,dt = -4$

$g(0) = \int_4^0 f(t)\,dt = 6$



$f'(x)$ is semicircle on $[2,6]$

Given $f(2) = 4$

① Where is $f(x)$ increasing?

$f' > 0$ $(-5,0) \cup (2,6)$

② $f(x)$ decreasing?

$f' < 0$ $(0,2)$

$\int_0^2 f'(x)\,dx = f(2) - f(0)$

$-1 = 4 - f(0)$

③ Determine $f(0)$: 5

$\int_{-5}^2 f'(x)\,dx = f(2) - f(-5)$

$4 = 4 - f(-5)$ $f(-5)$: 0

④ $\int_{-5}^6 f'(x)\,dx = 4 + 2\pi$

⑤ $\int_{-5}^6 2f'(x) - 8\,dx = 4\pi - 80$

$8 + 4\pi - 88$

⑥ $\int_{-5}^6 |f'(x)|\,dx = 6 + 2\pi$