Review L'Hopital's Rule and Improper Integrals (2016)

Evaluate the following integrals. Determine if the integral converges or diverges.

(1) $\int_0^1 \frac{1}{x}\,dx$ (2) $\int_1^\infty \frac{1}{x}\,dx$ (3) $\int_0^\infty xe^{-x}\,dx$

(4) $\int_0^\infty \frac{1}{1+x^2}\,dx$ (5) $\int_5^\infty \frac{1}{\sqrt{x-1}}\,dx$ (6) $\int_0^1 \frac{1}{1-x}\,dx$

(7) $\int_1^\infty \ln x\,dx$ (14) $\int_{-\infty}^\infty xe^{-x^2}\,dx$ (6) $\int_0^2 \frac{x}{1-x}\,dx$ (12) $\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}\,dx$

(22) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ (23) $\int_0^4 \frac{x}{\sqrt{16-x^2}}\,dx$ (27) $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

Evaluate the following limits. Show all work.

1. $\lim_{x\to 3} \frac{2x-6}{x^2-9}$ 2. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$ 3. $\lim_{x\to\infty} \frac{5x^2-3x+1}{3x^2-5}$

4. $\lim_{x\to 2} \frac{x^3-x-2}{x-2}$ 5. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x}$ 6. $\lim_{x\to 0} \frac{e^x-(1-x)}{x}$ 7. $\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$ 8. $\lim_{x\to 0} \frac{\arcsin x}{x}$

9. $\lim_{x\to\infty} \frac{3x^2-2x+1}{2x^2+3}$ 10. $\lim_{x\to\infty} \frac{x^2+2x+1}{x-1}$ 11. $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$ 12. $\lim_{x\to\infty} \frac{\ln x}{x}$ 13. $\lim_{x\to\infty} \frac{(\ln x)^3}{x}$

14. $\lim_{x\to 0^+} (-x\ln x)$ 15. $\lim_{x\to\infty} \left(x\sin\frac{1}{x}\right)$ 16. $\lim_{x\to\infty} x^{1/x}$ 17. $\lim_{x\to\infty} 4x^{1/x}$ 18. $\lim_{x\to\infty} (4x)^{1/x}$

19. $\lim_{x\to 0^+} (1+x)^{\frac{1}{6x}}$ 20. $\lim_{x\to 2^+} \left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$ 21. $\lim_{x\to 1^+} \left(\frac{3}{\ln x}-\frac{2}{x-1}\right)$ 22. $\lim_{x\to\infty} \frac{x^2}{e^{5x}}$

23. $\lim_{x\to 0} \frac{e^{2x}-1}{e^x}$

24. Find the area between $y=(x-8)^{\frac{-2}{3}}$ and y = 0 for [0, 8)

. Find the area of the region to the right of x = 1 between $y=\frac{2}{4x^2-1}$ and the x-axis.

Review L'Hopital's Rule and Improper Integrals (2015)

Key

Evaluate the following integrals. Determine if the integral converges or diverges.

(1) $\int_0^1 \frac{1}{x}\,dx = \infty$ diverges

(2) $\int_1^\infty \frac{1}{x}\,dx = \infty$ diverges

(3) $\int_0^\infty xe^{-x}\,dx = 1$ conv.

(4) $\int_0^\infty \frac{1}{1+x^2}\,dx$ $= \pi/2$ conv.

(5) $\int_5^\infty \frac{1}{\sqrt{x-1}}\,dx$ $= \infty$ diverges

(6) $\int_0^1 \frac{1}{1-x}\,dx = \infty$ diverges

(7) $\int_1^\infty \ln x\,dx$ $= \infty$ div.

(14) $\int_{-\infty}^\infty xe^{-x^2}\,dx$ $= 0$ conv.

(6) $\int_0^2 \frac{x}{1-x}\,dx \to$ split @ 1 $= \infty$ div.

(12) $\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}\,dx \to$ split! @ 2 $= 3+3\sqrt[3]{2}$ conv.

(22) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ $= \pi/2$ conv.

(23) $\int_0^4 \frac{x}{\sqrt{16-x^2}}\,dx$ $= 4$ conv.

(27) $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx \to$ split! Problems @ both ends! $= 2$ conv.

Evaluate the following limits. Show all work.

1. $\lim_{x\to 3} \frac{2x-6}{x^2-9} = 1/3$

2. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{1}{4}$

3. $\lim_{x\to \infty} \frac{5x^2-3x+1}{3x^2-5} = \frac{5}{3}$

4. $\lim_{x\to 2} \frac{x^3-x-2}{x-2}$ $=$ DNE

5. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x}$ $= 0$

6. $\lim_{x\to 0} \frac{e^x-(1-x)}{x}$ $= 2$

7. $\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$ $= 2/3$

8. $\lim_{x\to 0} \frac{\arcsin x}{x} = 1$

9. $\lim_{x\to \infty} \frac{3x^2-2x+1}{2x^2+3}$ $= \frac{3}{2}$

10. $\lim_{x\to \infty} \frac{x^2+2x+1}{x-1}$ $= \infty$

11. $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$ $= -1$

12. $\lim_{x\to \infty} \frac{\ln x}{x}$ $= 0$

13. $\lim_{x\to \infty} \frac{(\ln x)^3}{x}$ $= 0$

14. $\lim_{x\to 0^+} (-x\ln x)$ $= 0$

15. $\lim_{x\to \infty} \left(x\sin\frac{1}{x}\right) = 1$

16. $\lim_{x\to \infty} x^{1/x}$ $= 1$

17. $\lim_{x\to \infty} 4x^{1/x}$ $= 4$

18. $\lim_{x\to \infty} (4x)^{1/x} = 1$

19. $\lim_{x\to 0^+} (1+x)^{\frac{1}{6x}}$ $= e^{1/6}$

20. $\lim_{x\to 2^+} \left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$ $= -3/2$

21. $\lim_{x\to 1^+} \left(\frac{3}{\ln x}-\frac{2}{x-1}\right)$ $= \infty$

22. $\lim_{x\to \infty} \frac{x^2}{e^{5x}} = 0$

23. $\lim_{x\to 0} \frac{e^{2x}-1}{e^x} = 0$

24. Find the area between $y=(x-8)^{\frac{2}{3}}$ and y = 0 for [0, 8)

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25. Find the area of the region to the right of x = 1 between $y=\frac{2}{4x^2-1}$ and the x-axis.

$= \lim_{b\to\infty}\left[\frac{1}{2}\ln|2x-1| - \frac{1}{2}\ln|2x+1|\right]_1^b$

$= \lim_{b\to\infty}\left[\frac{1}{2}\ln\left|\frac{2b-1}{2b+1}\right| - \frac{1}{2}\ln\frac{1}{3}\right]$

$\boxed{\frac{1}{2}\ln 3}$

OR

$-\frac{1}{2}\ln\frac{1}{3}$