Integration Techniques - Review #2

1. $\int x\sqrt{x^2-1}\,dx$

2. $\int x\sqrt{x-1}\,dx$

3. $\int x\arcsin(5x^2)\,dx$

4. $\int \cos^3(3x-1)\,dx$

5. $\int \sin^2\left(\frac{x}{2}\right)dx$

6. $\int x^2\sin 2x\,dx$

7. $\int \frac{x^2}{x^2+2x-15}\,dx$

8. $\int \frac{4}{x^2+6x+10}\,dx$

9. $\int \frac{2x^3-5x^2+4x-4}{x^2-x}\,dx$

10. $\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

11. $\int \frac{e^x}{1+e^{2x}}\,dx$

12. $\int \frac{e^x}{1+e^x}\,dx$

13. $\int \frac{1}{\cos^2 3x}\,dx$

14. $\int \frac{1}{\sqrt{1-4x-x^2}}\,dx$

15. $\int \frac{2x-1}{x^2+2x+2}\,dx$

16. $\int \frac{4x+3}{\sqrt{1-x^2}}\,dx$

17. $\int \sec^4(5x)\,dx$

18. $\int \tan x\sec^4 x\,dx$

19. $\int x^4\ln(2x)\,dx$

20. $\int \arcsin x\,dx$

21. $\int \frac{1}{1+e^{-x}}\,dx$

22. $\int \frac{\cos^2 x}{\sin x}\,dx$

Key Int Rev II

$\frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C$

1. $\frac{1}{3}(x^2-1)^{3/2} + C$ 2. $\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

3. $\frac{1}{10}\left[5x^2 \arcsin(5x^2) + \sqrt{1-(5x^2)^2}\right] + C$ 4. $\frac{1}{3}\sin(3x-1) - \frac{1}{9}\sin^3(3x-1) + C$

5. $\frac{1}{2}x - \frac{1}{2}\sin x + C$ 6. $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

7. $x + \frac{9}{8}\ln|x-3| - \frac{25}{8}\ln|x+5| + C$ 8. $4\arctan(x+3) + C$

9. $x^2 - 3x + 4\ln|x| - 3\ln|x-1| + C$ 10. $\frac{1}{2}(\arcsin x)^2 + C$

11. $\arctan(e^x) + C$ 12. $\ln|1+e^x| + C$ 13. $\frac{1}{3}\tan(3x) + C$

14. $\arcsin\left(\frac{x+2}{\sqrt{5}}\right) + C$ 15. $\ln|x^2+2x+2| - 3\arctan(x+1) + C$

16. $-4\sqrt{1-x^2} + 3\arcsin x + C$ 17. $\frac{1}{5}\tan 5x + \frac{1}{15}\tan^3 5x + C$

18. $\frac{1}{4}\sec^4 x + C$ 19. $\frac{1}{5}x^5 \ln 2x - \frac{1}{25}x^5 + C$

OR $\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$

20. $x\arcsin x + \sqrt{1-x^2} + C$ 21. $\ln|1+e^x| + C$

OR

$x + \ln|1+e^{-x}| + C$

22. $\ln|\csc x - \cot x| + \cos x + C$

1. $u = x^2 - 1$

$du = 2x\,dx$

$\frac{1}{2}\int u^{1/2}\,du$

$\boxed{= \frac{1}{3}(x^2-1)^{3/2} + C}$

2. $u = x-1 \quad x = u+1$

$du = dx$

$\int (u+1)\sqrt{u}\,du$

$= \int u^{3/2} + u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$\boxed{= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C}$

OR

$\boxed{\frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C}$

$\frac{2}{15}(x-1)^{3/2}[3(x-1)+5] + C$

$\frac{2}{15}(x-1)^{3/2}[3x+2] + C$

$\frac{2}{15}(x-1)^{3/2}[5x - 2(x-1)]$

$[3x+2] + C$

→ Both simplify to

3. $u = 5x^2$

$du = 10x\,dx$

$\frac{1}{10}\int \arcsin u\,du$

$u = \arcsin x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} \quad v = x$

$= \frac{1}{10}\left[u\arcsin u - \int \frac{u}{\sqrt{1-u^2}}\,du\right]$

$\boxed{= \frac{1}{10}\left[5x^2\arcsin(5x^2) + \sqrt{1-(5x^2)^2}\right] + C}$

} could do $u = \arcsin(5x^2) \quad dv = x$

$du = \frac{10x}{\sqrt{1-(5x^2)^2}}\,dx \quad v = \frac{1}{2}$

$w = 1-u^2$

$dw = -2u\,du$

$-\frac{1}{2}\int \frac{1}{\sqrt{w}}$

$-\sqrt{1-u^2}$

4. $\int \cos(3x-1)\cos^2(3x-1)\,dx$

$= \int \cos(3x-1)(1-\sin^2(3x-1))\,dx$

$u = \sin(3x-1)$

$du = 3\cos(3x-1)\,dx$

$\frac{1}{3}\int (1-u^2)\,du$

$= \frac{1}{3}\left[u - \frac{1}{3}u^3\right] + C$

$\boxed{= \frac{1}{3}\sin(3x-1) - \frac{1}{9}(\sin^3(3x-1)) + C}$

5. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\sin^2 \frac{x}{2} = \frac{1}{2}(1-\cos x)$

$= \frac{1}{2}\int (1-\cos x)\,dx$

$\boxed{= \frac{1}{2}x - \frac{1}{2}\sin x + C}$

6.

	u	dv
+	x^2	$\sin 2x$
−	$2x$	$-\frac{1}{2}\cos 2x$
+	2	$-\frac{1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$

$\boxed{-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C}$

7. $x^2+2x-15 \overline{\big)\, x^2}$ $\quad 1$

$$\begin{array}{r} -x^2+2x+15 \\ \hline -2x+15 \end{array}$$

$$1+\int \frac{-2x+15}{x^2+2x-15}\,dx$$

$(x$

7. $\boxed{x+\frac{9}{8}\ln|x-3|-\frac{25}{8}\ln|x+5|+C}$

8. $$\int \frac{4}{x^2+6x+9+10-9}\,dx$$

$(x+3)^2+1$

$\boxed{=4\arctan(x+3)+C}$

9. $\boxed{x^2-3x+4\ln|x|-3\ln|x-1|+C}$

10. $u=\arcsin x$

$du=\frac{1}{\sqrt{1-x^2}}$ $\quad \int u\,du$

$\boxed{\frac{1}{2}(\arcsin x)^2+C}$

11. $u=e^x$

$du=e^x dx$

$\int \frac{1}{1+u^2}\,du$

$\boxed{=\arctan(e^x)+C}$

12. $u=1+e^x$

$du=e^x dx$

$\int \frac{1}{u}\,du$

$\boxed{\ln|1+e^x|+C}$

13. $\int \sec^2(3x)\,dx$

$\boxed{=\frac{1}{3}\tan(3x)+C}$

14. $1-4x-x^2$

$-(x^2+4x+4)+1+4$

$$\int \frac{1}{\sqrt{5-(x+2)^2}}\,dx$$

$\boxed{\arcsin\left(\frac{x+2}{\sqrt{5}}\right)+C}$

15. $u=x^2+2x+2$

$du=2x+2\,dx$

$$\int \frac{2x+2}{x^2+2x+2}\,dx-3\int \frac{1}{x^2+2x+2}$$

$x^2+2x+1+1$

$(x+1)^2+1$

$\boxed{\ln|x^2+2x+2|-3\arctan(x+1)+C}$

16. $$\int \frac{4x}{\sqrt{1-x^2}}\,dx+\int \frac{3}{\sqrt{1-x^2}}\,dx$$

$u=1-x^2$

$du=-2x\,dx$

$-2\int \frac{1}{\sqrt{u}}$

$\boxed{-4\sqrt{1-x^2}+3\arcsin x+C}$

17. $\int \sec^2(5x)(1+\tan^2 5x)\,dx$

$u=\tan 5x$

$du=5\sec^2 5x$ $\quad \frac{1}{5}\int 1+u^2\,du$

$=\frac{1}{5}\left[u+\frac{1}{3}u^3\right]+C$ $\quad \frac{1}{5}\tan 5x+\frac{1}{15}\tan^3 5x+$

18. $\int \sec^3 x \underbrace{\sec x \tan x}\, dx$

$u = \sec x \quad du = \uparrow$

$\int u^3 du = \boxed{\frac{1}{4}\sec^4 x + c}$

OR $\boxed{\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + c}$

19. $u = \ln 2x \quad dv = x^4 dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{5}x^5$

$\frac{1}{5}x^5 \ln 2x - \int \frac{1}{5}x^4 dx$

$\boxed{= \frac{1}{5}x^5 \ln 2x - \frac{1}{25}x^5 + c}$

20. $x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$

$\boxed{= x \arcsin x - \sqrt{1-x^2} + c}$

21. $\int \frac{e^x}{e^x + 1}$ $\boxed{\text{same as } \#12}$

OR $\int \frac{1+e^{-x}}{1+e^{-x}} - \int \frac{e^{-x}}{1+e^{-x}}$

$u = 1 + e^{-x}$

$-e^{-x}$

$\int 1 -$

$\boxed{x + \ln|1+e^{-x}| + c}$

22. $\int \frac{1-\sin^2 x}{\sin x} dx$

$= \int \csc x - \sin x$

$\boxed{= \ln|\csc x - \cot x| + \cos x + c}$