

$f(x) = 3x - 6$   $[1, 4]$  6 rect. Approx area under curve.

$$\Delta x = \frac{1}{2} = .5$$

L RAM =

$$\frac{1}{2}f(1) + \frac{1}{2}f(1.5) + \frac{1}{2}f(2) + \frac{1}{2}f(2.5) + \frac{1}{2}f(3) + \frac{1}{2}f(3.5) = \underline{2.25}$$

R RAM =

$$\frac{1}{2}f(1.5) + \frac{1}{2}f(2) + \frac{1}{2}f(2.5) + \frac{1}{2}f(3) + \frac{1}{2}f(3.5) + \frac{1}{2}f(4) = \underline{6.75}$$

M RAM =

$$\begin{aligned} &\frac{1}{2}f(1.25) + \frac{1}{2}f(1.75) + \frac{1}{2}f(2.25) + \frac{1}{2}f(2.75) \\ &+ \frac{1}{2}f(3.25) + \frac{1}{2}f(3.75) = \underline{4.5} \end{aligned}$$

$$\text{M RAM} \neq \frac{\text{L RAM} + \text{R RAM}}{2} \quad !!$$

$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i$  = actual "signed" area between  $f(x)$  and  $x$ -axis on  $[a, b]$

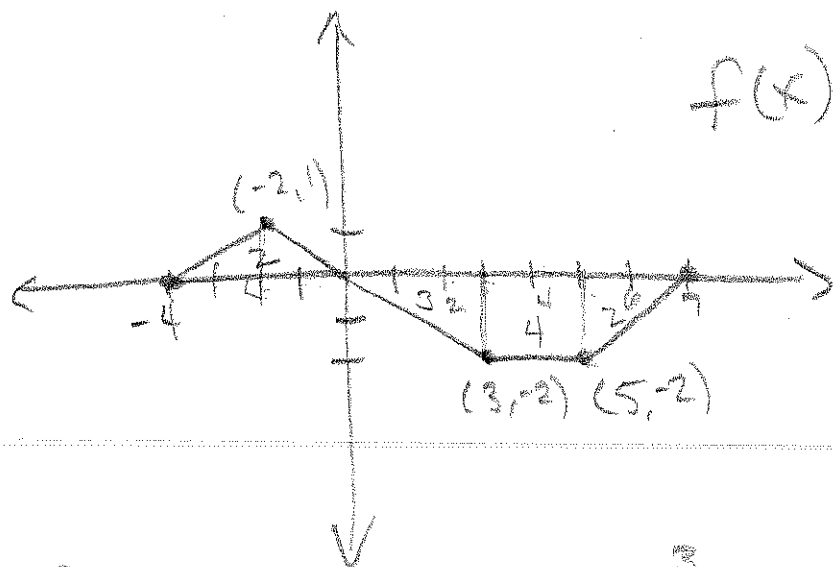
$\downarrow$   
 $\Delta x \rightarrow 0$

Riemann Sum  $\Rightarrow \int_a^b f(x) dx$

$b$  ← upper limit of integration  
 $a$  ← lower limit of integration

definite integral

If  $f(x) \leq 0$  on  $[a, b]$ ,  $\int_a^b f(x) dx \leq 0$



$$5. \int_{-4}^0 f(x) dx = \underline{-2}$$

$$6. \int_3^5 4f(x) dx = \underline{-16}$$

$$1. \int_{-4}^0 f(x) dx = \underline{2}$$

$$2. \int_0^3 f(x) dx = \underline{-3}$$

$$3. \int_0^7 f(x) dx = \underline{-9}$$

$$4. \int_{-4}^7 f(x) dx = \underline{-7}$$

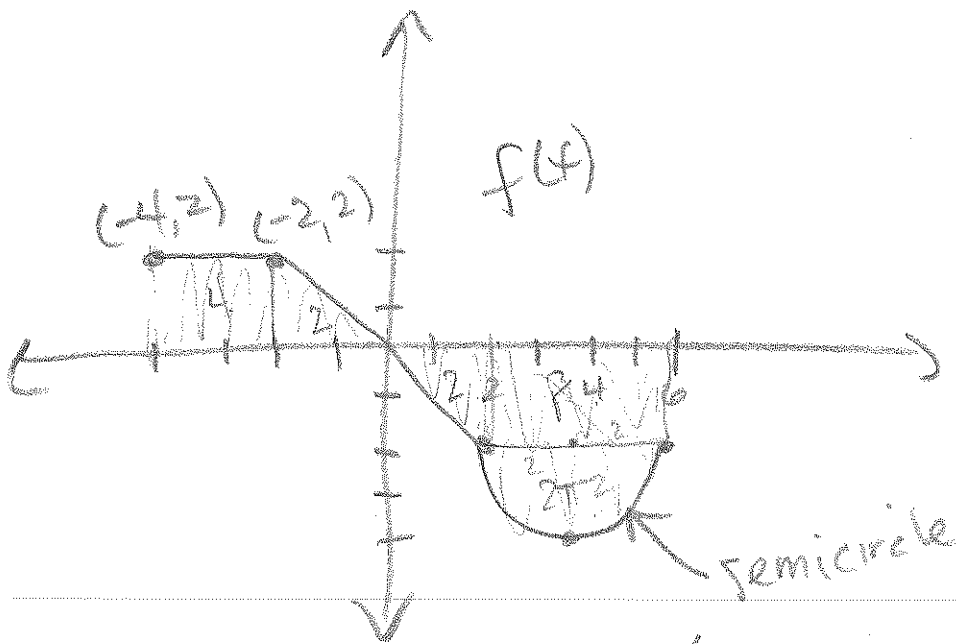
If  $c$  is between  $a$  and  $b$ ,  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b k f(x) dx = k \int_a^b f(x) dx$$

$$\int_a^b f(x) + g(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$\int_a^a f(x) dx = 0$$



$$\int_{-4}^0 f(x) dx = \underline{6}$$

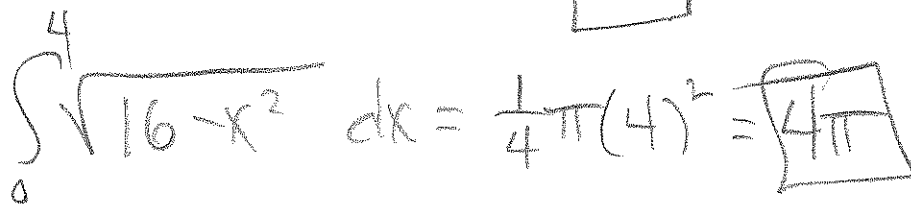
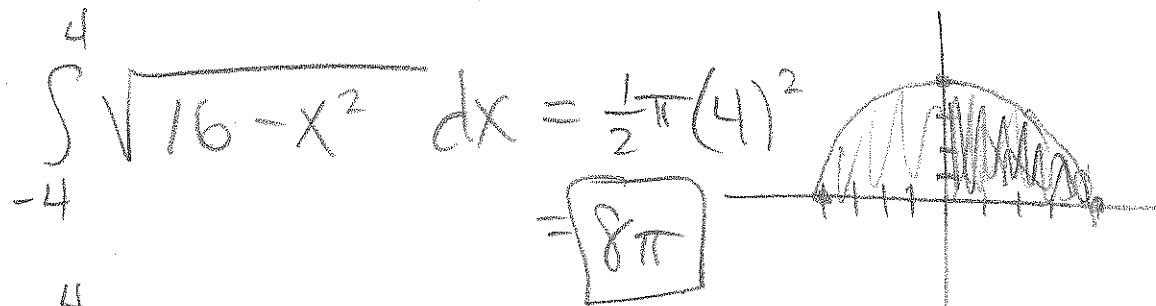
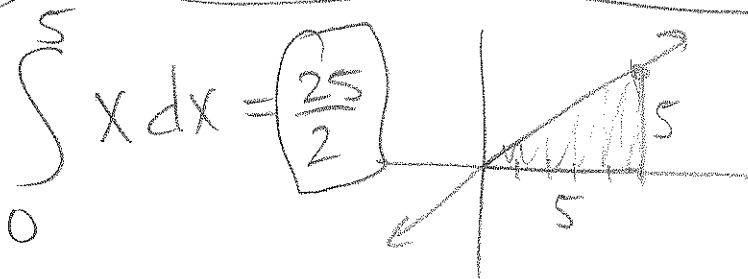
$$\int_{-2}^6 f(x) dx = \underline{-8 - 2\pi}$$

$$\int_{-4}^6 f(x) dx = \underline{-4 - 2\pi}$$

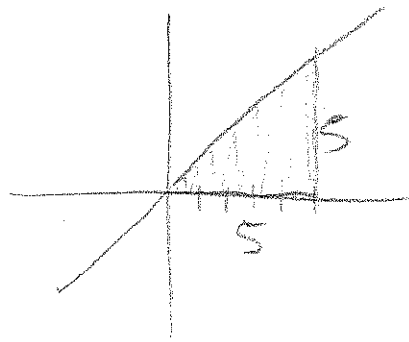
$$\int_6^{-4} f(x) dx = \underline{4 + 2\pi}$$

$$\int_0^2 4f(x) dx = \underline{-8}$$

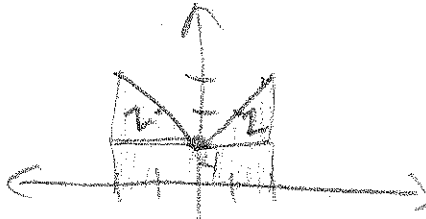
$$\int_{-4}^6 |f(x)| dx = \underline{16 + 2\pi}$$



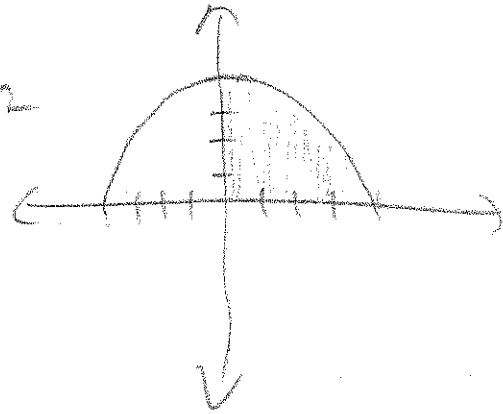
$$\int_0^5 x dx = 12.5$$



$$\int_{-2}^2 (|x|+1) dx = 8$$



$$\int_0^4 \sqrt{16-x^2} dx = \frac{1}{4} \pi (4)^2 = 4\pi$$



$$\int_3^6 5 dx = 15$$

$$\int_2^6 f(x) dx = 12$$

$$\int_2^4 f(x) dx = 8$$

a)  $\int_4^6 f(x) dx = \underline{4}$

b)  $\int_2^6 3f(x) dx = \underline{36}$

c)  $\int_4^2 f(x) dx = \underline{-8}$

d)  $\int_2^4 (f(x)+5) dx = \underline{18}$