Lab Series LAB 2

EXERCISES FOR SECTION 1.3

In Exercises 1–4, use a graphing utility to graph the function and visually estimate the limits.

1. $h(x) = x^2 - 5x$
   (a) $\lim_{x\to 5} h(x)$
   (b) $\lim_{x\to -1} h(x)$

2. $g(x) = \dfrac{12(\sqrt{x} - 3)}{x - 9}$
   (a) $\lim_{x\to 4} g(x)$
   (b) $\lim_{x\to 0} g(x)$

3. $f(x) = x \cos x$
   (a) $\lim_{x\to 0} f(x)$
   (b) $\lim_{x\to \pi/3} f(x)$

4. $f(t) = t|t - 4|$
   (a) $\lim_{t\to 4} f(t)$
   (b) $\lim_{t\to -1} f(t)$

In Exercises 5–22, find the limit.

5. $\lim_{x\to 2} x^4$
6. $\lim_{x\to -2} x^3$
7. $\lim_{x\to 0} (2x - 1)$
8. $\lim_{x\to -3} (3x + 2)$
9. $\lim_{x\to -3} (x^2 + 3x)$
10. $\lim_{x\to 1} (-x^2 + 1)$
11. $\lim_{x\to -3} (2x^2 + 4x + 1)$
12. $\lim_{x\to 1} (3x^3 - 2x^2 + 4)$
13. $\lim_{x\to 2} \dfrac{1}{x}$
14. $\lim_{x\to -3} \dfrac{2}{x + 2}$
15. $\lim_{x\to 1} \dfrac{x - 3}{x^2 + 4}$
16. $\lim_{x\to 3} \dfrac{2x - 3}{x + 5}$
17. $\lim_{x\to 7} \dfrac{5x}{\sqrt{x + 2}}$
18. $\lim_{x\to 3} \dfrac{\sqrt{x + 1}}{x - 4}$
19. $\lim_{x\to 3} \sqrt{x + 1}$
20. $\lim_{x\to 4} \sqrt[3]{x + 4}$
21. $\lim_{x\to -4} (x + 3)^2$
22. $\lim_{x\to 0} (2x - 1)^3$

In Exercises 23–26, find the limits.

23. $f(x) = 5 - x,\ g(x) = x^3$
    (a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 4} g(x)$ (c) $\lim_{x\to 1} g(f(x))$
24. $f(x) = x + 7,\ g(x) = x^2$
    (a) $\lim_{x\to -3} f(x)$ (b) $\lim_{x\to 4} g(x)$ (c) $\lim_{x\to -3} g(f(x))$
25. $f(x) = 4 - x^2,\ g(x) = \sqrt{x + 1}$
    (a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 3} g(x)$ (c) $\lim_{x\to 1} g(f(x))$
26. $f(x) = 2x^2 - 3x + 1,\ g(x) = \sqrt[3]{x + 6}$
    (a) $\lim_{x\to 4} f(x)$ (b) $\lim_{x\to 21} g(x)$ (c) $\lim_{x\to 4} g(f(x))$

In Exercises 27–36, find the limit of the trigonometric function.

27. $\lim_{x\to \pi/2} \sin x$
28. $\lim_{x\to \pi} \tan x$
29. $\lim_{x\to 2} \cos \dfrac{\pi x}{3}$
30. $\lim_{x\to 1} \sin \dfrac{\pi x}{2}$
31. $\lim_{x\to 0} \sec 2x$
32. $\lim_{x\to \pi} \cos 3x$
33. $\lim_{x\to 5\pi/6} \sin x$
34. $\lim_{x\to 5\pi/3} \cos x$
35. $\lim_{x\to 3} \tan\left(\dfrac{\pi x}{4}\right)$
36. $\lim_{x\to 7} \sec\left(\dfrac{\pi x}{6}\right)$

In Exercises 37–40, use the information to evaluate the limits.

37. $\lim_{x\to c} f(x) = 2$
    $\lim_{x\to c} g(x) = 3$
    (a) $\lim_{x\to c} [5g(x)]$
    (b) $\lim_{x\to c} [f(x) + g(x)]$
    (c) $\lim_{x\to c} [f(x)g(x)]$
    (d) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$

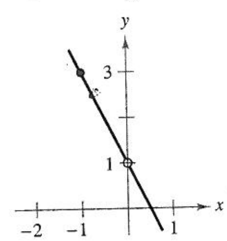
38. $\lim_{x\to c} f(x) = \frac{3}{2}$
    $\lim_{x\to c} g(x) = \frac{1}{2}$
    (a) $\lim_{x\to c} [4f(x)]$
    (b) $\lim_{x\to c} [f(x) + g(x)]$
    (c) $\lim_{x\to c} [f(x)g(x)]$
    (d) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$

39. $\lim_{x\to c} f(x) = 4$
    (a) $\lim_{x\to c} [f(x)]^3$
    (b) $\lim_{x\to c} \sqrt{f(x)}$
    (c) $\lim_{x\to c} [3f(x)]$
    (d) $\lim_{x\to c} [f(x)]^{3/2}$

40. $\lim_{x\to c} f(x) = 27$
    (a) $\lim_{x\to c} \sqrt[3]{f(x)}$
    (b) $\lim_{x\to c} \dfrac{f(x)}{18}$
    (c) $\lim_{x\to c} [f(x)]^2$
    (d) $\lim_{x\to c} [f(x)]^{2/3}$

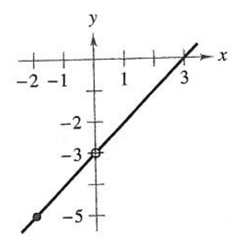
In Exercises 41–44, use the graph to determine the limit visually (if it exists). Write a simpler function that agrees with the given function at all but one point.

41. $g(x) = \dfrac{-2x^2 + x}{x}$



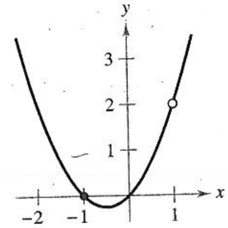
(a) $\lim_{x\to 0} g(x)$
(b) $\lim_{x\to -1} g(x)$

42. $h(x) = \dfrac{x^2 - 3x}{x}$



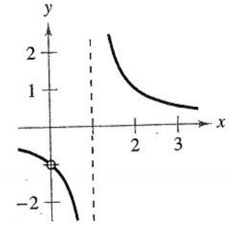
(a) $\lim_{x\to -2} h(x)$
(b) $\lim_{x\to 0} h(x)$

43. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim_{x\to 1} g(x)$
(b) $\lim_{x\to -1} g(x)$

44. $f(x) = \dfrac{x}{x^2 - x}$



(a) $\lim_{x\to 1} f(x)$
(b) $\lim_{x\to 0} f(x)$

In Exercises 45–48, find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

45. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

46. $\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$

47. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

48. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

In Exercises 49–62, find the limit (if it exists).

49. $\lim_{x \to 5} \frac{x - 5}{x^2 - 25}$

50. $\lim_{x \to 2} \frac{2 - x}{x^2 - 4}$

51. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$

52. $\lim_{x \to 4} \frac{x^2 - 5x + 4}{x^2 - 2x - 8}$

53. $\lim_{x \to 0} \frac{\sqrt{x + 5} - \sqrt{5}}{x}$

54. $\lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x}$

55. $\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4}$

56. $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}$

57. $\lim_{x \to 0} \frac{[1/(3 + x)] - (1/3)}{x}$

58. $\lim_{x \to 0} \frac{[1/(x + 4)] - (1/4)}{x}$

59. $\lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x}$

60. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

61. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 2(x + \Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

62. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$

Graphical, Numerical, and Analytic Analysis **In Exercises 63–66, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

63. $\lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x}$

64. $\lim_{x \to 16} \frac{4 - \sqrt{x}}{x - 16}$

65. $\lim_{x \to 0} \frac{[1/(2 + x)] - (1/2)}{x}$

66. $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$

In Exercises 67–78, determine the limit of the trigonometric function (if it exists).

67. $\lim_{x \to 0} \frac{\sin x}{5x}$

68. $\lim_{x \to 0} \frac{3(1 - \cos x)}{x}$

69. $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{2x^2}$

70. $\lim_{\theta \to 0} \frac{\cos \theta \tan \theta}{\theta}$

71. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

72. $\lim_{x \to 0} \frac{\tan^2 x}{x}$

73. $\lim_{h \to 0} \frac{(1 - \cos h)^2}{h}$

74. $\lim_{\phi \to \pi} \phi \sec \phi$

75. $\lim_{x \to \pi/2} \frac{\cos x}{\cot x}$

76. $\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

77. $\lim_{t \to 0} \frac{\sin 3t}{2t}$

78. $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$ $\left[\textit{Hint:} \text{ Find } \lim_{x \to 0}\left(\frac{2 \sin 2x}{2x}\right)\left(\frac{3x}{3 \sin 3x}\right).\right]$

Graphical, Numerical, and Analytic Analysis **In Exercises 79–82, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

79. $\lim_{t \to 0} \frac{\sin 3t}{t}$

80. $\lim_{h \to 0} (1 + \cos 2h)$

81. $\lim_{x \to 0} \frac{\sin x^2}{x}$

82. $\lim_{x \to 0} \frac{\sin x}{\sqrt[3]{x}}$

In Exercises 83–86, find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$.

83. $f(x) = 2x + 3$

84. $f(x) = \sqrt{x}$

85. $f(x) = \frac{4}{x}$

86. $f(x) = x^2 - 4x$

In Exercises 87 and 88, use the Squeeze Theorem to find $\lim_{x \to c} f(x)$.

87. $c = 0$

$4 - x^2 \le f(x) \le 4 + x^2$

88. $c = a$

$b - |x - a| \le f(x) \le b + |x - a|$

In Exercises 89–94, use a graphing utility to graph the given function and the equations $y = |x|$ **and** $y = -|x|$ **in the same viewing window. Using the graphs to visually observe the Squeeze Theorem, find** $\lim_{x \to 0} f(x)$.

89. $f(x) = x \cos x$

90. $f(x) = |x \sin x|$

91. $f(x) = |x| \sin x$

92. $f(x) = |x| \cos x$

93. $f(x) = x \sin \frac{1}{x}$

94. $h(x) = x \cos \frac{1}{x}$

Getting at the Concept

95. In the context of finding limits, discuss what is meant by two functions that agree at all but one point.

96. Give an example of two functions that agree at all but one point.

97. What is meant by an indeterminate form?

98. In your own words, explain the Squeeze Theorem.

99. ***Writing*** Use a graphing utility to graph

$$f(x) = x, \quad g(x) = \sin x, \quad \text{and} \quad h(x) = \frac{\sin x}{x}$$

in the same viewing window. Compare the magnitudes of $f(x)$ and $g(x)$ when x is "close to" 0. Use the comparison to write a short paragraph explaining why

$$\lim_{x \to 0} h(x) = 1.$$