More Practice 3.3 & Quiz Review

Find all critical numbers. Then determine the intervals where $f(x)$ is increasing/decreasing. Classify x-values of local extrema.

1. $f(x) = x^4 - 8x^2 + 1$

2. $f(x) = 10x^3(x-1)^2$

3. $f(x) = x(x-5)^{1/3}$

4. $f(x) = x\sqrt{4-x^2}$

5. $f(x) = x + 2\cos x \quad [0, 2\pi]$

6. $f(x) = 2\cos x + \cos 2x \quad [0, 2\pi]$

7. $f(x) = \dfrac{4}{x-2}$

8. Determine <u>absolute</u> extreme values of $f(x) = (x-1)^{2/3} + 2 \quad [0, 9$

Determine if Rolle's Thm applies & if so, find c to satisfy conclusion

9. $f(x) = x^4 + 4x^2 + 1 \quad [-3, 3]$

10. $f(x) = 5 + 3(x-1)^{2/3} \quad [0, 2]$

Determine if Mean Value Thm applies, and if so, find c to satisfy conclusion.

11. $f(x) = x + \dfrac{4}{x} \quad [1, 4]$

12. $f(x) = 1 - 3x^{1/3} \quad [-8, -1]$

1. CN: $-2, 0, 2$ $\downarrow (-\infty,-2) \cup (0,2)$ $\uparrow (-2,0) \cup (2,\infty)$
LMax: $x=0$ LMin: $x=\pm 2$

2. CN: $0, \frac{3}{5}, 1$ $\downarrow (\frac{3}{5}, 1)$ $\uparrow (-\infty, \frac{3}{5}) \cup (1,\infty)$ $x=3/5$ Max $x=1$ M

3. CN: $\frac{15}{4}, 5$ $\downarrow (-\infty, \frac{15}{4})$ $\uparrow (\frac{15}{4}, 5) \cup (5,\infty)$ $x=\frac{15}{4}$ min

4. CN: $\pm 2, \pm\sqrt{2}$ $\downarrow (-2,-\sqrt{2}) \cup (\sqrt{2}, 2)$ $\uparrow (-\sqrt{2}, \sqrt{2})$ $x=-\sqrt{2}$ min $x=\sqrt{2}$ ma

5. CN: $\frac{\pi}{6}, \frac{5\pi}{6}$ $\downarrow (\frac{\pi}{6}, \frac{5\pi}{6})$ $\uparrow (0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$ $x=\pi/6$ max $x=\frac{5\pi}{6}$ min

6. CN: $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$ $\downarrow (0, \frac{2\pi}{3}) \cup (\pi, \frac{4\pi}{3})$ $\uparrow (\frac{2\pi}{3}, \pi) \cup (\frac{4\pi}{3}, 2\pi)$
$x=\pi$ max $x=\frac{2\pi}{3}, \frac{4\pi}{3}$ min

7. CN: None $\downarrow (-\infty, 2) \cup (2,\infty)$ $\uparrow$ Never No extrema

8. Abs max $= 6$ @ $x=9$ Abs min $= 2$ @ $x=1$

9. ① $f(x)$ is cont on $[-3,3]$ ② $f'(x) = 4x^3+8x$ is cont on $(-3,3)$ so $f(x)$ is diff $(-3,3)$
③ $f(-3) = 118 = f(3)$ ∴ Rolle's applies $4x^3+8x=0$
$4x(x^2+2)=0$ $x=0$
$\boxed{c=0}$

10. ① $f(x)$ is cont on $[0,2]$ ② $f'(x) = \frac{2}{(x-1)^{1/3}}$ is not cont @ $x=1$
so $f(x)$ is NOT diff on $(0,2)$ ∴ Rolle's Thm does not apply

11. ① $f(x) = x + \frac{4}{x}$ is cont on $[1,4]$ ② $f'(x) = 1 - \frac{4}{x^2}$ is cont $(1,4)$ so f is diff $(1,4)$
∴ MVT applies $f' = 1 - \frac{4}{x^2} = \frac{5-5}{4-1} \Rightarrow 1 - \frac{4}{x^2} = 0$ $1 = \frac{4}{x^2}$ $x^2 = 4$
$x = \pm 2$
$\boxed{c=2}$

12. ① $f(x)$ is cont on $[-8,-1]$ ② $f' = -x^{-2/3} = \frac{-1}{x^{2/3}}$ is cont on $(-8,-1)$ so f diff $(-8,-1)$
MVT applies
$\frac{-1}{x^{2/3}} = \frac{4-7}{-1--8} \Rightarrow \frac{-1}{x^{2/3}} = \frac{-3}{7} \Rightarrow x^{2/3} = \frac{7}{3}$ $x = \pm(\frac{7}{3})^{3/2}$
$\boxed{c = -(7/3)^{3/2}}$