Volume if rotating around horizontal or vertical lines <u>not</u> x or y-axis:

* Remember, <u>radius</u> is the length from center to pt on outside, so if line of rotation is different than axis, radius is length from curve to the line. Radius "values" should always be positive.

Disk = <u>No</u> space. Line rotating around is a boundary of the region

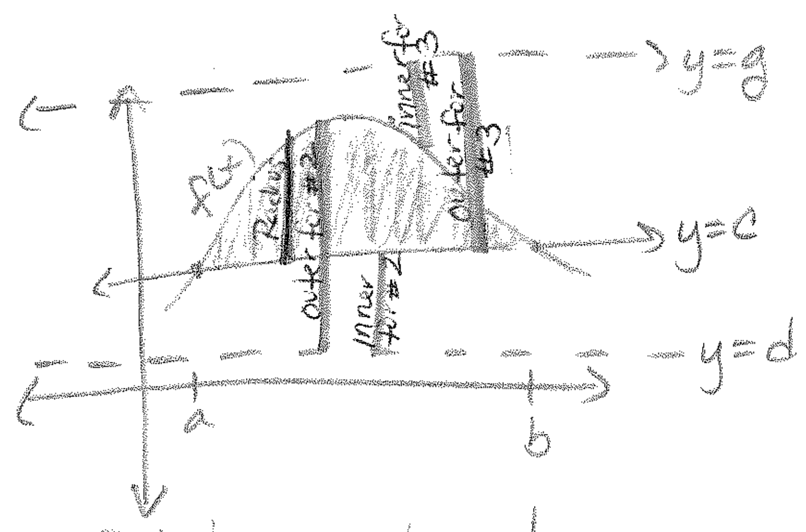
Washer = space → <u>hollow</u>.

Horiz line ⇒ y = # in terms of x

Vert line ⇒ x = # in terms of y.

Disk ⇒ $\pi \int_a^b (\text{radius})^2 \, dx$ or dy

Washer = $\pi \int_a^b \left(\begin{matrix}\text{outer}\\\text{radius}\end{matrix}\right)^2 - \left(\begin{matrix}\text{inner}\\\text{radius}\end{matrix}\right)^2 dx$ or dy



* Region bounded by $y=f(x)$ and $y=c$.

① Rotate around $y=c$
* In terms of <u>x</u> <u>Disk</u>

$$\pi \int_a^b (\underbrace{f(x)-c}_{\text{length of rectangle}})^2 dx$$

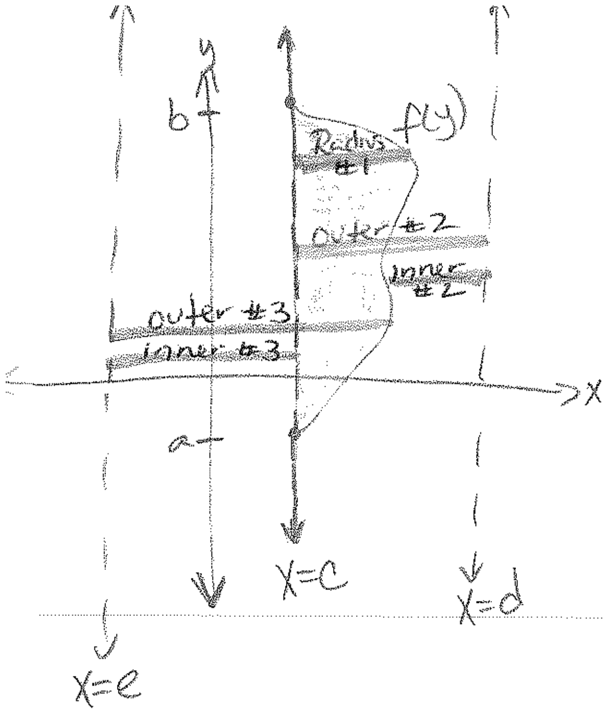
② Rotate around $y=d$
* In terms of <u>x</u> <u>Washer</u>

$$\pi \int_a^b (f(x)-d)^2 - (c-d)^2 \, dx$$

* would <u>not</u> matter if d was below x-axis

③ Rotate around $y=g$
* In terms of <u>x</u> Washer

$$\pi \int_a^b (g-c)^2 - (g-f(x))^2 \, dx$$



* Region bounded by f(y) and X=c.

① Rotate around X=c.

* In terms of y Disk

$$\pi\int_a^b (f(y)-c)^2\,dy$$

② Rotate around X=d

* (y) Washer

$$\pi\int_a^b (d-c)^2-(d-f(y))^2\,dy$$

③ Rotate around X=e

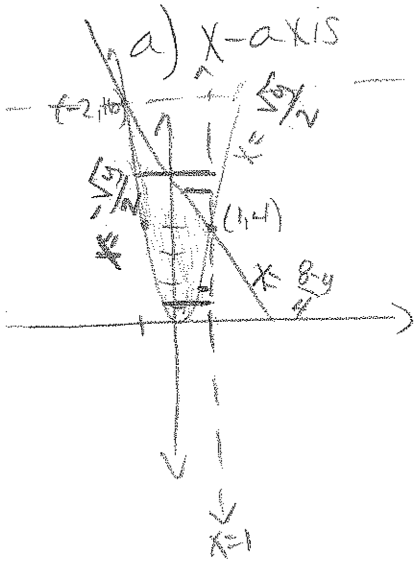
* (y) Washer

$$\pi\int_a^b (f(y)-e)^2-(c-e)^2\,dy$$

Example:

Region bounded by $y=4x^2$ and $4x+y=8$ rotated around

a) X-axis b) X=1 c) y=16



a) (X) Washer ⇒ $\pi\int_{-2}^{1} (8-4x)^2-(4x^2)^2\,dx = \frac{1152}{5}\pi$

b) (y) Washer ⇒ 2 regions

$$\pi\int_0^4 \left(1-\frac{\sqrt{y}}{2}\right)^2-\left(1-\frac{\sqrt{y}}{2}\right)^2 dy$$

$$+\ \pi\int_4^{16} \left(1-\frac{\sqrt{y}}{2}\right)^2-\left(1-\frac{8-y}{4}\right)^2 dy = 54\pi$$

c) (X) Washer

$$\pi\int_{-2}^{1} (16-4x^2)^2-(16-(8-4x))^2\,dx = \frac{1728}{5}\pi$$