# Separable Differential Eqs

* Given implicitly (x + y mixed up) or deriv. is embedded in equation

→ <u>separate</u> (get $y'$ by itself) + then rewrite $\boxed{f(y)dy = f(x)dx}$

- Integrate both sides + use any initial conditions to find constant values.
- solve for y (usually)

* If <u>not</u> separable, approx values using Euler's Method

Ex. $\frac{dy}{dx} = 3x^2y^3$

$\int \frac{1}{y^3}dy = \int 3x^2\,dx$

$-\frac{1}{2y^2} = x^3 + c$

$\frac{1}{y^2} = -2(x^3+c) = -2x^3 - 2c$

$y^2 = \frac{1}{-2x^3-2c}$

$\boxed{y = \pm\sqrt{\frac{1}{-2x^3-2c}}}$ ⇒ general solution

Ex. $\frac{dy}{dx} = x + y$

$dy = (x+y)dx$

$\frac{dy}{dx} - y = x$

* <u>Not</u> separable.

$$y' = \frac{4x}{y^2} \Rightarrow \frac{dy}{dx} = \frac{4x}{y^2}$$

$$\int y^2\,dy = \int 4x\,dx$$

$$\frac{1}{3}y^3 = 2x^2 + c$$

$$y^3 = 6x^2 + 3c$$

$$y = \sqrt[3]{6x^2+3c} \rightarrow \text{general soln.}$$

x y

(1, 2) ⇒ on soln curve

"particular soln" $\Rightarrow \frac{1}{3}(8) = 2(1)^2 + c$

$$c = \frac{8}{3} - 2 = \frac{2}{3}$$

$$\boxed{y = \sqrt[3]{6x^2+2}}$$

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$$4xy' = \cos^2 y \Rightarrow y' = \frac{\cos^2 y}{4x} \Rightarrow \frac{dy}{dx} = \frac{\cos^2 y}{4x}$$

$$\int \frac{1}{\cos^2 y}\,dy = \int \frac{1}{4x}\,dx$$

$$\int \sec^2 y\,dy = \frac{1}{4}\int \frac{1}{x}\,dx$$

$$\tan y = \frac{1}{4}\ln|x| + c$$

$$y = \arctan\left(\frac{1}{4}\ln|x| + c\right)$$

$$\cancel{\tan 0} = \frac{1}{4}\ln 3 + c \qquad c = -\frac{1}{4}\ln 3$$

$$y = \arctan\left(\frac{1}{4}\ln|x| - \frac{1}{4}\ln 3\right)$$

$$\boxed{y = \arctan\left(\frac{1}{4}\ln\left|\frac{x}{3}\right|\right)}$$

$$xy + y' = 100x \Rightarrow y' = 100x - xy$$

$$\frac{dy}{dx} = x(100-y)$$

$$\int \frac{1}{100-y}\,dy = \int x\,dx$$

$u = 100-y$

$du = -dy$

$-\int \frac{1}{u}\,du$

$$-\ln|100-y| = \frac{1}{2}x^2 + c$$

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$$\ln|100-y| = -\frac{1}{2}x^2 - c$$

$$e \qquad e$$

not same

$c$

$$|100-y| = e^{-\frac{1}{2}x^2 - c}$$

$$|100-y| = Ce^{-\frac{1}{2}x^2}$$

$* \; C = e^{-c}$

$$100-y = \pm Ce^{-\frac{1}{2}x^2}$$

$$\boxed{y = 100 \mp Ce^{-\frac{1}{2}x^2}}$$

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Ex: $\dfrac{dP}{dt} = kP$

variables $\Rightarrow$ $P$ and $t$

$k \Rightarrow$ constant

$$\int \frac{1}{P}\,dP = \int k\,dt$$

* separate & integrate
* Solve for $P \Rightarrow$ exponentiate
* Use values to find $C$ and $k$

$$\ln|P| = kt + c$$

$$e \qquad e$$

$1000 = Ce^{k(0)}$ $\quad C = 1000$

$$P = Ce^{kt} \longrightarrow (A = Pe^{rt})$$

$P = 1000e^{kt}$

$1200 = 1000e^{k(2)}$ $\quad \ln 1.2 = 2k$

$1.2 = e^{2k}$ $\quad k = \underline{\ln 1.2}$

a) $P = 1000e^{(\frac{1}{2}\ln 1.2)t}$

$P = 1000e^{.09116t}$

b) Plug in 10 for t

$2488.32 \approx 2488$ people

Ex. $\frac{dV}{dt} = kV$

$\int \frac{1}{V}\,dV = \int k\,dt$

$\ln|V| = kt + C$

$e^{\ln|V|} = e^{kt+C}$

$V = Ce^{kt}$

$t=0 \Rightarrow V=500$

$t=10 \quad V=400$

$500 = Ce^0 \Rightarrow C = 500$

$400 = 500e^{10k}$

$\frac{4}{5} = e^{10k} \Rightarrow k = \frac{\ln 4/5}{10}$

$$\boxed{\begin{gathered} V = 500e^{(\frac{1}{10}\ln\frac{4}{5})t} \\ V = 500e^{-.0223t} \end{gathered}}$$

b) $50 = 500e^{-.0223t}$

$.1 = e^{-.0223t}$

$t = \frac{\ln .1}{-.0223} = \boxed{103.1885 \text{ sec}}$

halflife $\Rightarrow \frac{\ln 1/2}{K}$