AP Calculus - Review for Series Test

Determine whether sequence converges or diverges.

$a_n = \frac{n+1}{n^2}$ 2. $a_n = \left(1+\frac{1}{2n}\right)^n$ 3. $a_n = \frac{\sin\sqrt{n}}{\sqrt{n}}$

Determine the convergence or divergence of the series. If it converges find the sum

4. $\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n$ 5. $\sum_{n=0}^{\infty}\frac{2^{n+2}}{3^n}$ 6. $\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)$

7. $\sum_{n=0}^{\infty}\left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)}\right]$ 8. $\sum_{n=1}^{\infty}\frac{2n+1}{3n+1}$

If series is positive termed, determine if convergent or divergent. If series contains negative terms, determine if absolutely convergent, conditionally convergent, or divergent.

7. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n(n+1)(n+2)}}$ 8. $\sum_{n=0}^{\infty}\frac{(2n+3)^2}{(n+1)^3}$ 9. $\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^{n+1}$ 10. $\sum_{n=0}^{\infty}\frac{1}{2+\left(\frac{1}{2}\right)^n}$

11. $\sum_{n=1}^{\infty}\frac{3^{2n+1}}{n5^{n-1}}$ 12. $\sum_{n=1}^{\infty}\frac{1}{3^n+2}$ 13. $\sum_{n=1}^{\infty}\frac{n!}{\ln(n+1)}$ 14. $\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}$

15. $\sum_{n=1}^{\infty}(n^2+9)(-2)^{1-n}$ 16. $\sum_{n=1}^{\infty}\frac{n+\cos n}{n^3+1}$ 17. $\sum_{n=1}^{\infty}\frac{e^n}{n^e}$ 18. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$

19. $\sum_{n=2}^{\infty}(-1)^n\frac{\sqrt[3]{n-1}}{n^2-1}$ 20. $\sum_{n=1}^{\infty}(-1)^n\frac{2n+3}{n!}$ 21. $\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$ 22. $\sum_{n=1}^{\infty}\frac{(2n)^n}{n^{2n}}$

23. $\sum_{n=1}^{\infty}\frac{e^{2n}}{(2n-1)!}$ 24. $\sum_{n=1}^{\infty}\left(\frac{1}{3^n}-\frac{5}{\sqrt{n}}\right)$

1. $\lim_{n\to\infty} \frac{n+1}{n^2} = 0$ $\boxed{\text{conv. to } 0}$

2. $\lim_{n\to\infty} \left(1+\frac{1}{2n}\right)^n = e^{1/2}$ $\boxed{\text{Conv to } e^{1/2}}$

3. $\lim_{n\to\infty} \frac{\sin\sqrt{n}}{\sqrt{n}} = 0$ $\boxed{\text{Conv. to } 0}$

4. Geo $r = \frac{2}{3} < 1$ so conv. Sum $= \frac{1}{1-\frac{2}{3}} = \boxed{3}$

5. $\sum_{n=0}^{\infty} 4\left(\frac{2}{3}\right)^n$ Geo $r = \frac{2}{3} < 1$ so conv.

Sum $= \frac{4}{1-\frac{2}{3}} = \boxed{12}$

6. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n = 2 - \frac{3}{2} = \boxed{\frac{1}{2}}$

Geo $r = \frac{1}{2} < 1$ conv

sum $= \frac{1}{1-\frac{1}{2}} = 2$

Geo $r = \frac{1}{3} < 1$ conv

Sum $= \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

7. $\sum\left(\frac{2}{3}\right)^n \Rightarrow$ Geo $r = \frac{2}{3} < 1$ conv

Sum $= \frac{1}{1-\frac{2}{3}} = 3$

$\sum \frac{1}{(n+1)(n+2)} \Rightarrow$ telescoping conv.

Sum $= 1$

$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots$

$\Rightarrow 3 - 1 = \boxed{2}$

8. $\lim_{n\to\infty} \frac{2n+1}{3n+1} = \frac{2}{3} \neq 0$ div. by n^{th} term test

$\boxed{\text{Bottom \#7}}$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n(n+1)(n+2)}} = a_n$ $\quad b_n = \frac{1}{n}$ div b/c harmonic

$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n(n+1)(n+2)}} \cdot \frac{n}{1} = 1 > 0$ $\therefore$ By lim. comp, since $\sum b_n$ div, $\sum a_n$ div.

$\boxed{\text{Bottom \#8}}$ $\sum_{n=0}^{\infty} \frac{(2n+3)^2}{(n+1)^3} = a_n$ $\quad b_n = \frac{n^2}{n^3} = \frac{1}{n}$ div b/c harmonic

$\lim_{n\to\infty}\left(\frac{(2n+3)^2}{(n+1)^3} \cdot \frac{n}{1}\right) = 4 > 0$ $\therefore$ By LC, since $\sum b_n$ div, $\sum a_n$ div. too.

9. Geo $|r| = \left|-\frac{2}{3}\right| < 1$ so <u>conv.</u> <u>absolutely</u>

10. $\lim_{n\to\infty} \frac{1}{2+\left(\frac{1}{2}\right)^n} = \frac{1}{2} \neq 0$

$\therefore$ Div. by n^{th} term test

11. $\lim_{n\to\infty} \left| \frac{3^{2n+3}}{(n+1)5^n} \cdot \frac{n5^{n-1}}{3^{2n+1}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{3^{2n}} \cdot \overset{9}{\cancel{3^3}} \cdot n \cdot \cancel{5^n} \cdot 5^{-1}}{(n+1)\cancel{5^n} \cdot \cancel{3^{2n}} \cdot \cancel{3}} \right|$

$= \lim_{n\to\infty} \left| \frac{9n}{5(n+1)} \right| = \frac{9}{5} > 1$ $\therefore$ By ratio test $\sum a_n$ <u>div.</u>

12. $\sum \frac{1}{3^n+2} = a_n$ $\quad b_n = \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$

Geo $r = \frac{1}{3} < 1$ conv.

$\frac{1}{3^n+2} \leq \frac{1}{3^n}$ $\therefore$ By direct comp, since $\sum b_n$ conv $\sum a_n$ conv.

13. $\lim_{n\to\infty} \frac{n!}{\ln(n+1)} = \infty \neq 0$ $\therefore$ Div by n^{th} term

14. $\lim_{n\to\infty} \frac{n^2-1}{n^2+1} = 1 \neq 0$ $\therefore$ Div by n^{th} term

15. $\sum_{n=1}^{\infty} (-1)^{1-n}(n^2+9)(2)^{1-n} = a_n$ $\quad a_{n+1} = (-1)^{-n}\left((n+1)^2+9\right)(2)^{-n}$

$\lim_{n\to\infty} \left| \frac{(n+1)^2+9}{\cancel{2^n}} \cdot \frac{\overset{\cancel{2^n}\cdot 2^{-1}}{\cancel{2^{n-1}}}}{n^2+9} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2+9}{2(n^2+9)} \right| = \frac{1}{2} < 1$ $\therefore$ By ratio test, $\sum a_n$ <u>conv.</u> <u>abs.</u>

24. $\sum\left(\frac{1}{3}\right)^n \Rightarrow$ Geo $r = \frac{1}{3} < 1$ conv

$\sum 5 \cdot \frac{1}{\sqrt{n}} \Rightarrow$ p series $p = \frac{1}{2} < 1$ div.

Conv − div = $\boxed{\text{diverges}}$

16. $\sum_{n=1}^{\infty} \frac{n+\cos n}{n^3+1} = a_n \quad b_n = \frac{n}{n^3} = \frac{1}{n^2}$ p series $p=2>1$ so conv.

$\lim_{n\to\infty}\left(\frac{n+\cos n}{n^3+1}\cdot\frac{n^2}{1}\right) = 1 > 0$ ∴ By limit comp, since $\sum b_n$ conv $\sum a_n$ conv. also.

(# bet 1 & −1)

17. $\lim_{n\to\infty} \frac{e^n}{n^e} = \infty \neq 0$ ∴ Div by n^{th} term test

(exp, poly)

18. Abs conv? $\sum \left|(-1)^{n-1}\frac{n}{n^2+1}\right| = \sum \frac{n}{n^2+1} = a_n \quad b_n = \frac{1}{n}$ div. harmonic

$\lim_{n\to\infty} \frac{n}{n^2+1}\cdot\frac{n}{1} = 1 > 0$ ∴ By lim. comp. since $\sum b_n$ diverges $\sum |a_n|$ diverges so not abs conv.

Cond? $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$ ✓

$f'(n) = \frac{n^2+1-n(2n)}{(n^2+1)^2} = \frac{1-n^2}{(n^2+1)^2} < 0$ when $n>1$ so decr.

∴ By AST, $\sum (-1)^{n-1}\frac{n}{n^2+1}$ is conditionally conv.

19. $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt[3]{n-1}}{n^2-1} = a_n \quad b_n = \frac{\sqrt[3]{n}}{n^2} = \frac{1}{n^{5/3}}$ p series $p=\frac{5}{3}>1$ so conv.

$\lim_{n\to\infty}\left(\frac{\sqrt[3]{n-1}}{n^2-1}\cdot\frac{n^{5/3}}{1}\right) = 1 > 0$ ∴ By LC, $\sum |a_n|$ conv since $\sum b_n$ conv so $\sum a_n$ is absolutely conv.

20. $\lim_{n\to\infty}\left|\frac{2(n+1)+3}{\cancel{(n+1)!}_{(n+1)}}\cdot\frac{\cancel{n!}}{2n+3}\right| = 0 < 1$ ∴ By ratio test, $\sum a_n$ is absolutely conv.

21. $\sum \frac{\cos n}{n^2} = a_n \quad b_n = \frac{1}{n^2}$ p series $p=2>1$ conv.

$\frac{\cos n}{n^2} \le \frac{1}{n^2}$ ∴ By direct comp, since $\sum b_n$ conv $\sum a_n$ is abs. conv.

22. $\lim_{n\to\infty}\left(\frac{(2n)^n}{n^{2n}}\right)^{1/n} = \lim_{n\to\infty}\left(\frac{2n}{n^2}\right) = 0 < 1$ ∴ By root test, $\sum a_n$ conv.

23. $\lim_{n\to\infty}\left|\frac{e^{2n+2}}{(2n+1)!}\cdot\frac{(2n-1)!}{e^{2n}}\right| = \lim_{n\to\infty}\left|\frac{e^2}{(2n+1)(2n)}\right| = 0 < 1$ ∴ By ratio test, $\sum a_n$ conv.