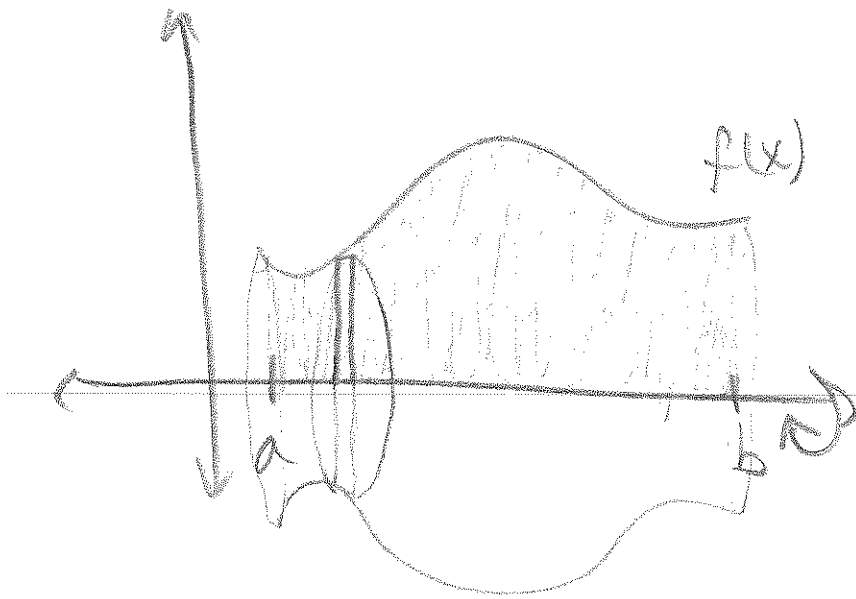


Volumes of Rotation



Disk

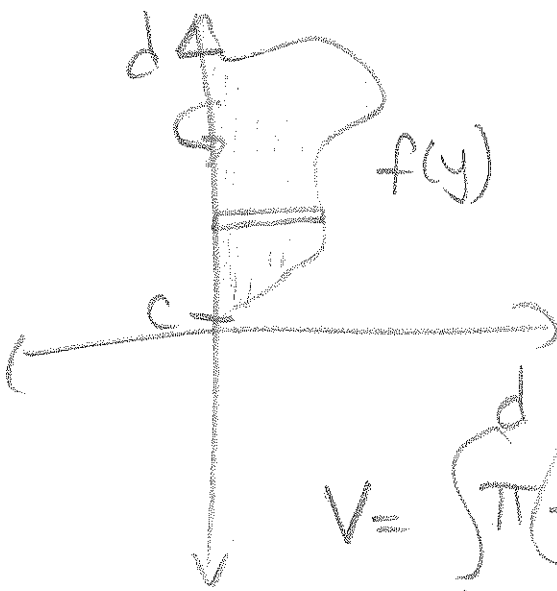
$$V = \pi r^2 h$$

↑ thickness (Δx)

$$\pi (f(x))^2 \Delta x$$

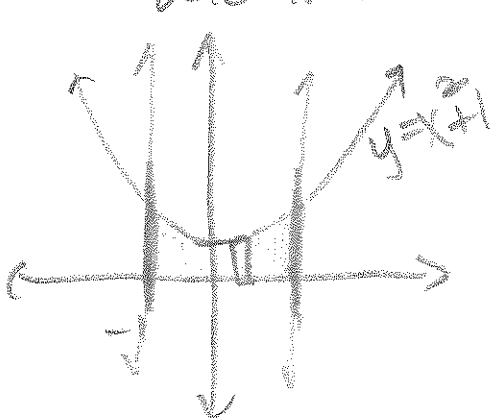
$$V = \int_a^b \pi (f(x))^2 dx$$

$$V = \pi \int_a^b (f(x))^2 dx$$



$$V = \int_c^d \pi (f(y))^2 dy = \pi \int_c^d (f(y))^2 dy$$

Ex. The region bounded by x-axis, $y = x^2 + 1$, $x = -1$, and $x = 1$ is rotated about x-axis. Find volume.



(X)

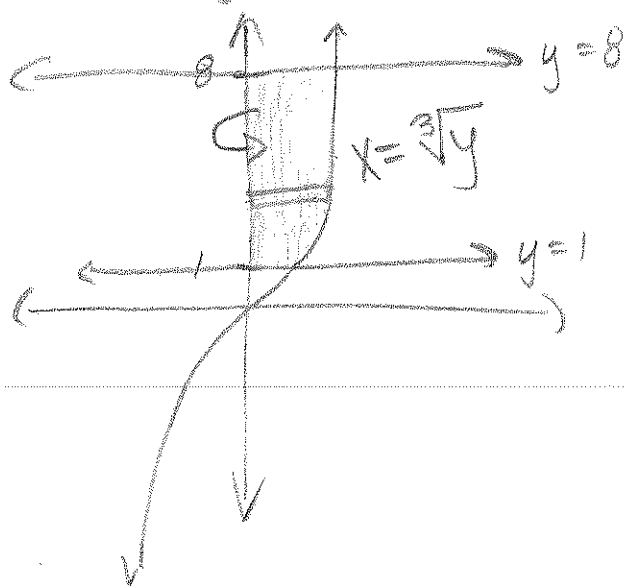
Disk

$$\pi \int_{-1}^1 (x^2 + 1)^2 dx$$

$$= 3.73\pi$$

$$= \underline{56\pi} = 11.729$$

The region bounded by y-axis, $y = x^3$, $y = 1$, and $y = 8$ is rotated about y-axis. Find volume.

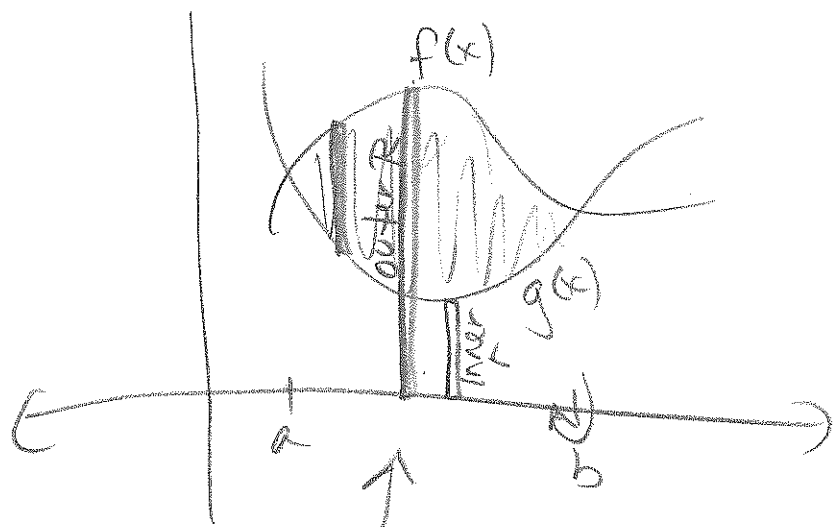


(4) Disk

$$\pi \int_1^8 (\sqrt[3]{y})^2 dy$$

$$= 58.4336$$

$$= 18.6\pi = \frac{93\pi}{5}$$



Washer

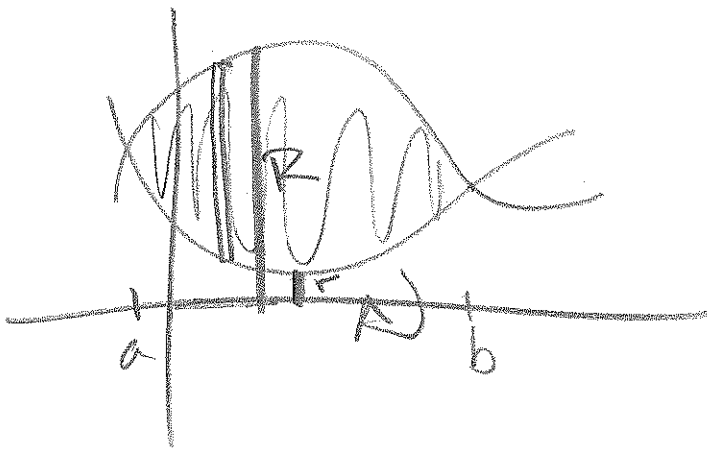
$$V_{\text{outer}} - V_{\text{inner space}}$$

$$\pi (\text{outer radius})^2 h - \pi (\text{inner radius})^2 h$$

$$\pi (f(x))^2 \Delta x - \pi (g(x))^2 \Delta x$$

* Hollow

$$V = \pi \int_a^b (f(x))^2 - (g(x))^2 dx$$



Washer \rightarrow hollow



$$V_{\text{outer}} - V_{\text{inner (hollow)}}$$

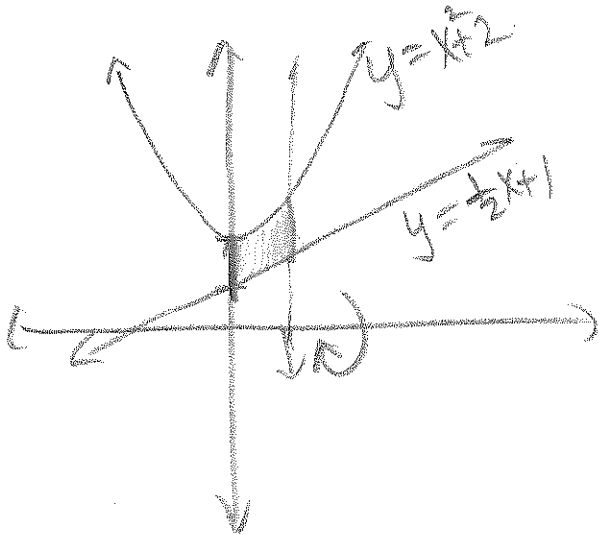
$$\int_a^b \pi R^2 dx - \int_a^b \pi r^2 dx$$

$$\pi \int_a^b (R^2 - r^2) dx$$

$$\pi \int_a^b (\text{outer})^2 - (\text{inner})^2 dx$$

$$x^2 = y - 2 \quad 2y - x - 2 = 0 \quad x=0 \quad x=1$$

Rotated around x-axis.



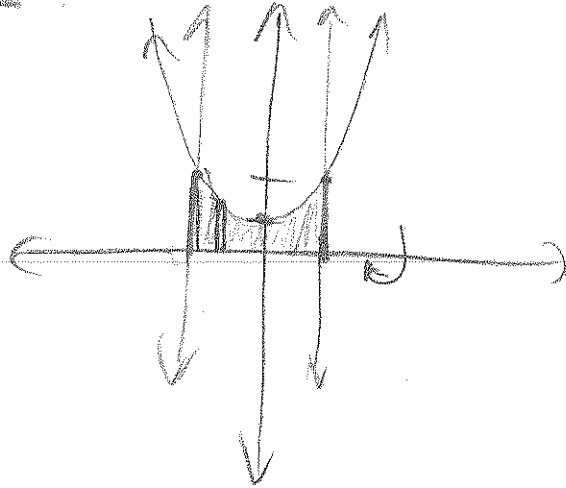
⊗ Washer

$$\pi \int_0^1 (x^2 + 2)^2 - \left(\frac{1}{2}x + 1\right)^2 dx$$

P. 429 # 1-9 odd
23-37 odd

The region bounded by x -axis, $y = x^2 + 1$, $x = -1$, and $x = 1$ is rotated about x -axis. Find volume

(X)



Disk

$$\pi \int_{-1}^1 (x^2 + 1)^2 dx$$

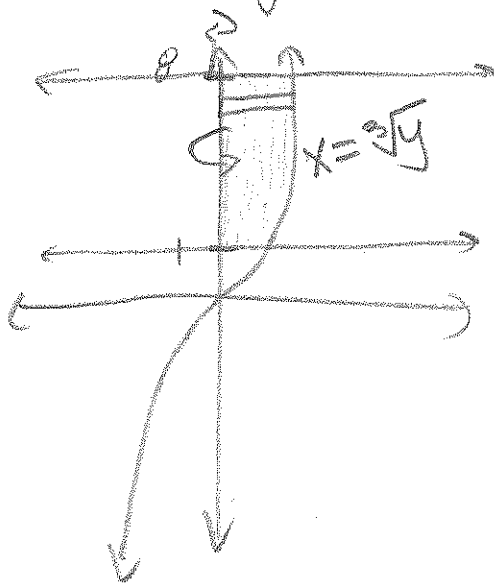
$$= \frac{56}{15} \pi$$

$$= 3.73\pi$$

$$= 11.7286$$

units³

The region bounded by y -axis, $y = x^3$, $y = 1$, and $y = 8$ is rotated about y -axis. Find volume



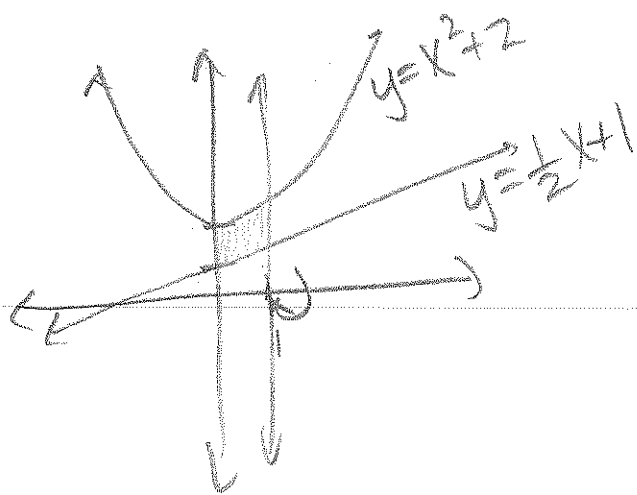
(Y)

$$\pi \int_1^8 (\sqrt[3]{y})^2 dy$$

$$= 18.6\pi$$

$$= \frac{93}{5} \pi = 58.4336$$

The region bounded by $x^2 = y - 2$ and $2y - x - 2 = 0$, and $x = 0$ and $x = 1$ is rotated around X-axis.



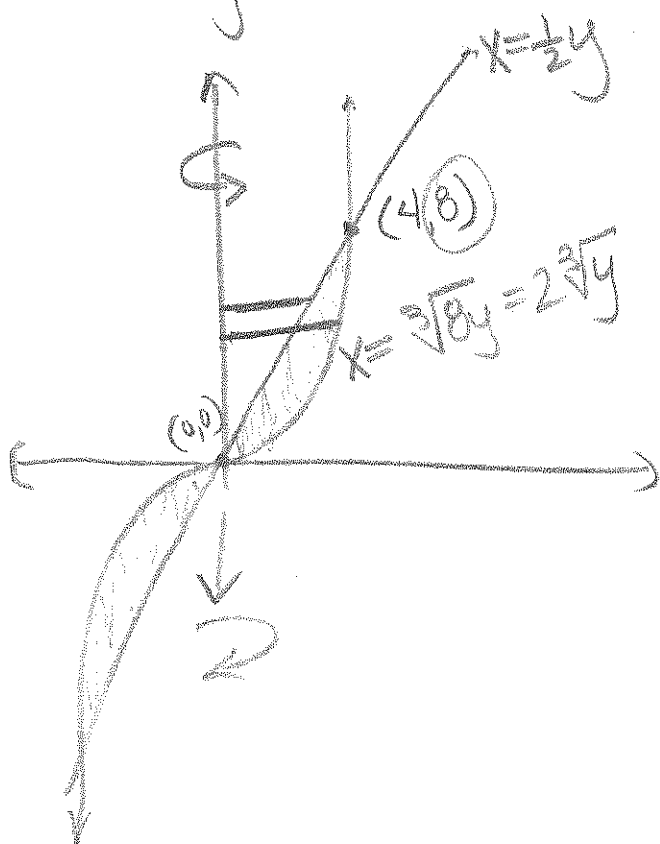
(X) Washer

$$\pi \int_0^1 (x^2 + 2)^2 - \left(\frac{1}{2}x + 1\right)^2 dx$$

$$= 12.409$$

$$= 3.95\pi$$

The region in 1st quadrant bounded by $y = \frac{1}{8}x^3$ and $y = 2x$ is rotated about y-axis. Find volume.



(y) Washer

$$\pi \int_0^8 (2\sqrt[3]{y})^2 - \left(\frac{1}{2}y\right)^2 dy$$

$$= 34.133\pi$$

$$= 107.233$$

P. 429 #1-9 odd, 23-37 odd

