3 types of tangent line problems 9/8/14

① Given f(x) and a pt (or x value) on f(x).

Ex. $f(x) = x^2 + 2x - 1$. Find equation of tangent at $x = 2$.

$$\lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - 1 - (x^2 + 2x - 1)}{h} = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 2x + 2h - 1 - x^2 - 2x + 1}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 2)}{h} = 2x + 2$$

$f'(2) = 6$ $\quad \boxed{y - 7 = 6(x - 2)}$

$(2, 7)$

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② Given f(x) and given slope you want (parallel, ⊥, horiz)

Take deriv & set = to what want + find x.

Ex. $f(x) = x^2 + 2x - 1$. Find equation of tangent to $f(x)$ that is parallel to $y = 4x - 3$.

$$f'(x) = 2x + 2 = 4$$
$$2x = 2$$
$$x = 1$$

$m = 4$ $\quad \boxed{y - 2 = 4(x - 1)}$

$(1, 2)$

Where is the tangent horizontal? Set $f' = 0$

$$2x + 2 = 0$$
$$x = -1$$

$(-1, -2)$

③ Given f(x) and a pt. <u>not</u> on f(x) but tangent line <u>passes</u> <u>through</u> it

* still need to find x-values on f(x) !!

Ex. $f(x)=x^2+2x-1$. Find equations of all tangents to f(x) that pass through (1,1).

$f'(x)=2x+2$

pt. on curve (x, x^2+2x-1)
+ given pt. $(1,1)$

slope formula $\frac{y_2-y_1}{x_2-x_1}$

$$2x+2=\frac{x^2+2x-1-1}{x-1}$$

$$(2x+2)(x-1)=x^2+2x-2$$

$$2x^2-2=x^2+2x-2$$

$$x^2-2x=0$$

$$x(x-2)=0$$

	$x=0$	$x=2$
$f'(x)=2x+2$	$m=2$ $(1,1)$	$m=6$ $(1,1)$
	$\boxed{y-1=2(x-1)}$	$\boxed{y-1=6(x-1)}$

TRY: $f(x)=4x-x^2$. Find equation of tangent that passes through $(2,5)$. TYPE 3!

$$\lim_{h\to 0}\frac{4(x+h)-(x+h)^2-(4x-x^2)}{h}=\lim_{h\to 0}\frac{\cancel{4x}+4h-\cancel{x^2}-2hx-h^2-\cancel{4x}+\cancel{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(4-2x-h)}{\cancel{h}}=4-2x$$

$f'(x)=4-2x$

$(2,5)\quad(x,\,4x-x^2)$

$$4-2x=\frac{4x-x^2-5}{x-2}$$

$$(4-2x)(x-2)=4x-x^2-5$$

$$4x-2x^2+4x-8=4x-x^2-5$$

$$-2x^2+8x-8=4x-x^2-5$$

$$0=x^2-4x+3$$

$$0=(x-3)(x-1)$$

	$x=3$	$x=1$
$f'=4-2x$	$m=-2$	$m=2$
	$(2,5)$	$(2,5)$
	$\boxed{y-5=-2(x-2)}$	$\boxed{y-5=2(x-2)}$

Derivative Rules

Constant Rule: $\frac{d}{dx}[c] = 0$

Linear Rule: $\frac{d}{dx}[mx+b] = m$

** Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$

$$\frac{d}{dx}[x^3] = 3x^2 \qquad \frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{1/2}] = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left[\frac{1}{x^4}\right] = \frac{d}{dx}[x^{-4}] = -4x^{-5} = \frac{-4}{x^5}$$

Sum & Difference Rule $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

$$\frac{d}{dx}[x^5 - x^3 + 7x - 8] = 5x^4 - 3x^2 + 7$$

Constant Mult. Rule $\frac{d}{dx}[cf(x)] = cf'(x)$

Ex. $\frac{d}{dx}[4x^5] = 4 \cdot 5x^4 = \boxed{20x^4}$

$$\frac{d}{dx}\left[4x^2 - \frac{3}{x^3} + \sqrt[3]{x^2}\right] = \frac{d}{dx}[4x^2 - 3x^{-3} + x^{2/3}]$$

$$= 8x + 9x^{-4} + \frac{2}{3}x^{-1/3} = 8x + \frac{9}{x^4} + \frac{2}{3\sqrt[3]{x}}$$

$$\frac{d}{dx}[5x^2(2x-8)] = \frac{d}{dx}[10x^3 - 40x^2] = 30x^2 - 80x$$