9/21/18

Higher order derivatives

* Take derivatives of derivatives

2nd deriv: y'' OR $\frac{d^2y}{dx^2}$ ⇒ take deriv. of y'

3rd: y'''

4th: $y^{(4)}$ ← parentheses tell you deriv # not power

$y = 3x^3 - 2x^2 + 7x - 1$ $\quad y''$?

$y' = 9x^2 - 4x + 7$

$y'' = 18x - 4$ $\quad y''' = 18$ $\quad y^{(4)} = 0$

$y = (3x^2 - 4x + 1)^3 = f(g(x))$ $\quad g(x) = 3x^2 - 4x + 1 = u$

$f(x) = x^3 \Rightarrow u^3$

$y = \sin\sqrt{x} = f(g(x))$ $\quad g(x) = \sqrt{x} \Rightarrow u$

$f(x) = \sin x \Rightarrow \sin u$

Book ⇒ "u" = the stuff

Chain Rule: $\frac{d}{dx}[f(g(x))] = g'(x) \cdot f'(g(x))$

"the deriv of inside stuff times the deriv of outside of the orig stuff"

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

① $y = (3x^2-4x+1)^3$ (stuff)

$y' = (6x-4) \cdot 3(3x^2-4x+1)^2$ (stuff)

(deriv of stuff)

$u = 3x^2-4x+1$

$y = u^3$

$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$

$\frac{dy}{dx} = 3u^2(6x-4)$

$= 3(3x^2-4x+1)^2(6x-4)$

② $y = \sin(\sqrt{x})$ (stuff)

$y' = \frac{1}{2}x^{-1/2} \cdot \cos\sqrt{x}$ (stuff) $= \frac{\cos\sqrt{x}}{2\sqrt{x}}$

(deriv of stuff)

③ $y = \sec(5x^3-2x+7)$ (stuff)

$y' = (15x^2-2)\sec(5x^3-2x+7)\tan(5x^3-2x+7)$

(deriv of stuff) (deriv of sec(stuff))

④ $y = \sin^2 x = (\sin x)^2$ (stuff)

$y' = \cos x \cdot 2(\sin x) = 2\sin x\cos x = \sin 2x$ (stuff)

(deriv stuff)

⑤ $y = \sin x^2$ → Note difference between this & last prob.

$y' = (2x)\cos(x^2) = 2x\cos x^2$

(deriv stuff)

(6.) $y = \left(\frac{3x^3-2}{5x+1}\right)^4$ → stuff ⇒ use quotient rule

$y' = \left(\frac{9x^2(5x+1)-(3x^3-2)(5)}{(5x+1)^2}\right) \cdot 4\left(\frac{3x^3-2}{5x+1}\right)^3$

(deriv. of stuff) · (stuff)

(7) $y = \sqrt{4x^3\tan x} = (4x^3\tan x)^{1/2}$ (stuff; use prod rule)

$y' = (12x^2\tan x + 4x^3\sec^2 x) \cdot \frac{1}{2}(4x^3\tan x)^{-1/2}$ (deriv of stuff; stuff)

$= \frac{2\cancel{4}x^2(3\tan x + x\sec^2 x)}{\cancel{2}\sqrt{4x^3\tan x}} = \frac{2x^2(3\tan x + x\sec^2 x)}{\sqrt{4x^3\tan x}}$

8. $y = \sec^3(\sqrt{5x-3}) = (\sec\sqrt{5x-3})^3$ Chain in ! chain ∘ in chain

$y = (\sec(5x-3)^{1/2})^3$

$y' = 5 \cdot \frac{1}{2}(5x-3)^{-1/2} \cdot \sec(5x-3)^{1/2}\tan(5x-3)^{1/2} \cdot 3(\sec(5x-3)^{1/2})^2$

5 ↑ deriv of $5x-3$; $\frac{1}{2}(5x-3)^{-1/2}$ ↑ deriv of $(\quad)^{1/2}$; $\sec(5x-3)^{1/2}\tan(5x-3)^{1/2}$ deriv of $\sec(\quad)$; $3(\sec(5x-3)^{1/2})^2$ deriv of $(\quad)^3$