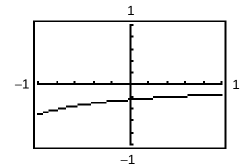
Review Exercises for Chapter 1

1. Calculus required. Using a graphing utility, you can estimate the length to be 8.3.
Or, the length is slightly longer than the distance between the two points, 8.25.

3.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-0.26	-0.25	-0.250	-0.2499	-0.249	-0.24

$\lim_{x\to 0} f(x) \approx -0.25$



5. $h(x) = \dfrac{x^2 - 2x}{x}$

(a) $\lim_{x\to 0} h(x) = -2$

(b) $\lim_{x\to -1} h(x) = -3$

7. $\lim_{x\to 1} (3 - x) = 3 - 1 = 2$

Let $\epsilon > 0$ be given. Choose $\delta = \epsilon$. Then for $0 < |x - 1| < \delta = \epsilon$, you have

$$|x - 1| < \epsilon$$
$$|1 - x| < \epsilon$$
$$|(3 - x) - 2| < \epsilon$$
$$|f(x) - L| < \epsilon$$

9. $\lim_{x\to 2} (x^2 - 3) = 1$

Let $\epsilon > 0$ be given. We need $|x^2 - 3 - 1| < \epsilon \Rightarrow |x^2 - 4| = |(x - 2)(x + 2)| < \epsilon \Rightarrow |x - 2| < \dfrac{1}{|x + 2|}\epsilon$.

Assuming, $1 < x < 3$, you can choose $\delta = \epsilon/5$. Hence, for $0 < |x - 2| < \delta = \epsilon/5$ you have

$$|x - 2| < \frac{\epsilon}{5} < \frac{1}{|x + 2|}\epsilon$$
$$|x - 2||x + 2| < \epsilon$$
$$|x^2 - 4| < \epsilon$$
$$|(x^2 - 3) - 1| < \epsilon$$
$$|f(x) - L| < \epsilon$$

11. $\lim_{t\to 4} \sqrt{t + 2} = \sqrt{4 + 2} = \sqrt{6} \approx 2.45$

13. $\lim_{t\to -2} \dfrac{t + 2}{t^2 - 4} = \lim_{t\to -2} \dfrac{1}{t - 2} = -\dfrac{1}{4}$

15. $$\lim_{x\to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x\to 4} \frac{\sqrt{x} - 2}{\left(\sqrt{x} - 2\right)\left(\sqrt{x} + 2\right)}$$
$$= \lim_{x\to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$$

17. $$\lim_{x\to 0} \frac{[1/(x + 1)] - 1}{x} = \lim_{x\to 0} \frac{1 - (x + 1)}{x(x + 1)}$$
$$= \lim_{x\to 0} \frac{-1}{x + 1} = -1$$

19. $$\lim_{x\to -5} \frac{x^3 + 125}{x + 5} = \lim_{x\to -5} \frac{(x + 5)(x^2 - 5x + 25)}{x + 5}$$
$$= \lim_{x\to -5} (x^2 - 5x + 25)$$
$$= 75$$

21. $\lim_{x\to 0} \dfrac{1 - \cos x}{\sin x} = \lim_{x\to 0} \left(\dfrac{x}{\sin x}\right)\left(\dfrac{1 - \cos x}{x}\right) = (1)(0) = 0$

23. $\lim_{\Delta x\to 0}\frac{\sin[(\pi/6)+\Delta x]-(1/2)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sin(\pi/6)\cos\Delta x+\cos(\pi/6)\sin\Delta x-(1/2)}{\Delta x}$

$$=\lim_{\Delta x\to 0}\frac{1}{2}\cdot\frac{(\cos\Delta x-1)}{\Delta x}+\lim_{\Delta x\to 0}\frac{\sqrt{3}}{2}\cdot\frac{\sin\Delta x}{\Delta x}$$

$$=0+\frac{\sqrt{3}}{2}(1)=\frac{\sqrt{3}}{2}$$

25. $\lim_{x\to c}[f(x)\cdot g(x)]=\left(-\frac{3}{4}\right)\left(\frac{2}{3}\right)=-\frac{1}{2}$

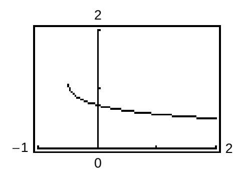
27. $f(x)=\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}$

(a)

x	1.1	1.01	1.001	1.0001
$f(x)$	0.5680	0.5764	0.5773	0.5773

$\lim_{x\to 1^+}\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}\approx 0.577$ (Actual limit is $\sqrt{3}/3$.)

(b)



(c) $\lim_{x\to 1^+}\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}=\lim_{x\to 1^+}\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}\cdot\frac{\sqrt{2x+1}+\sqrt{3}}{\sqrt{2x+1}+\sqrt{3}}$

$$=\lim_{x\to 1^+}\frac{(2x+1)-3}{(x-1)\left(\sqrt{2x+1}+\sqrt{3}\right)}$$

$$=\lim_{x\to 1^+}\frac{2}{\sqrt{2x+1}+\sqrt{3}}$$

$$=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

29. $\lim_{t\to a}\frac{s(a)-s(t)}{a-t}=\lim_{t\to 4}\frac{(-4.9(4)^2+200)-(-4.9t^2+200)}{4-t}$

$$=\lim_{t\to 4}\frac{4.9(t-4)(t+4)}{4-t}$$

$$=\lim_{t\to 4}-4.9(t+4)=-39.2\text{ m/sec}$$

31. $\lim_{x\to 3^-}\frac{|x-3|}{x-3}=\lim_{x\to 3^-}\frac{-(x-3)}{x-3}=-1$

33. $\lim_{x\to 2}f(x)=0$

35. $\lim_{t\to 1}h(t)$ does not exist because $\lim_{t\to 1^-}h(t)=1+1=2$ and $\lim_{t\to 1^+}h(t)=\frac{1}{2}(1+1)=1$.

37. $f(x)=[\![x+3]\!]$

$\lim_{x\to k^+}[\![x+3]\!]=k+3$ where k is an integer.

$\lim_{x\to k^-}[\![x+3]\!]=k+2$ where k is an integer.

Nonremovable discontinuity at each integer k

Continuous on $(k, k+1)$ for all integers k

39. $f(x)=\frac{3x^2-x-2}{x-1}=\frac{(3x+2)(x-1)}{x-1}$

$\lim_{x\to 1}f(x)=\lim_{x\to 1}(3x+2)=5$

Removable discontinuity at $x=1$

Continuous on $(-\infty, 1)\cup(1,\infty)$

41. $f(x)=\frac{1}{(x-2)^2}$

$\lim_{x\to 2}\frac{1}{(x-2)^2}=\infty$

Nonremovable discontinuity at $x=2$

Continuous on $(-\infty, 2)\cup(2,\infty)$

43. $f(x)=\frac{3}{x+1}$

$\lim_{x\to 1^-}f(x)=-\infty$

$\lim_{x\to 1^+}f(x)=\infty$

Nonremovable discontinuity at $x=-1$

Continuous on $(-\infty,-1)\cup(-1,\infty)$

45. $f(x) = \csc \frac{\pi x}{2}$

Nonremovable discontinuities at each even integer. Continuous on

$(2k, 2k + 2)$

for all integers k.

47. $f(2) = 5$

Find c so that $\lim_{x \to 2^+} (cx + 6) = 5$.

$$c(2) + 6 = 5$$
$$2c = -1$$
$$c = -\frac{1}{2}$$

49. f is continuous on $[1, 2]$. $f(1) = -1 < 0$ and $f(2) = 13 > 0$. Therefore by the Intermediate Value Theorem, there is at least one value c in $(1, 2)$ such that $2c^3 - 3 = 0$.

51. $f(x) = \frac{x^2 - 4}{|x - 2|} = (x + 2)\left[\frac{x - 2}{|x - 2|}\right]$

(a) $\lim_{x \to 2^-} f(x) = -4$

(b) $\lim_{x \to 2^+} f(x) = 4$

(c) $\lim_{x \to 2} f(x)$ does not exist.

53. $g(x) = 1 + \frac{2}{x}$

Vertical asymptote at $x = 0$

55. $f(x) = \frac{8}{(x - 10)^2}$

Vertical asymptote at $x = 10$

57. $\lim_{x \to -2^-} \frac{2x^2 + x + 1}{x + 2} = -\infty$

59. $\lim_{x \to -1^+} \frac{x + 1}{x^3 + 1} = \lim_{x \to -1^+} \frac{1}{x^2 - x + 1} = \frac{1}{3}$

61. $\lim_{x \to 1^-} \frac{x^2 + 2x + 1}{x - 1} = -\infty$

63. $\lim_{x \to 0^+} \left(x - \frac{1}{x^3}\right) = -\infty$

65. $\lim_{x \to 0^+} \frac{\sin 4x}{5x} = \lim_{x \to 0^+} \left[\frac{4}{5}\left(\frac{\sin 4x}{4x}\right)\right] = \frac{4}{5}$

67. $\lim_{x \to 0^+} \frac{\csc 2x}{x} = \lim_{x \to 0^+} \frac{1}{x \sin 2x} = \infty$

69. $C = \frac{80{,}000p}{100 - p},\ 0 \le 0 < 100$

(a) $C(15) \approx \$14{,}117.65$ (b) $C(50) = \$80.000$

(c) $C(90) = \$720{,}000$ (d) $\lim_{p \to 100^-} \frac{80{,}000p}{100 - p} = \infty$

Problem Solving for Chapter 1

1. (a) Perimeter $\triangle PAO = \sqrt{x^2 + (y - 1)^2} + \sqrt{x^2 + y^2} + 1$

$$= \sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1$$

Perimeter $\triangle PBO = \sqrt{(x - 1)^2 + y^2} + \sqrt{x^2 + y^2} + 1$

$$= \sqrt{(x - 1)^2 + x^4} + \sqrt{x^2 + x^4} + 1$$

(b) $r(x) = \frac{\sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1}{\sqrt{(x - 1)^2 + x^4} + \sqrt{x^2 + x^4} + 1}$

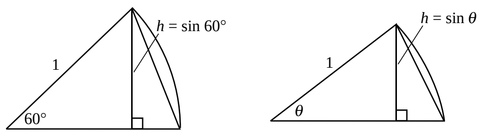
x	4	2	1	0.1	0.01
Perimeter $\triangle PAO$	33.02	9.08	3.41	2.10	2.01
Perimeter $\triangle PBO$	33.77	9.60	3.41	2.00	2.00
$r(x)$	0.98	0.95	1	1.05	1.005

(c) $\lim_{x \to 0^+} r(x) = \frac{1 + 0 + 1}{1 + 0 + 1} = \frac{2}{2} = 1$

3. (a) There are 6 triangles, each with a central angle of $60° = \pi/3$. Hence,

$$\text{Area hexagon} = 6\left[\frac{1}{2}bh\right] = 6\left[\frac{1}{2}(1)\sin\frac{\pi}{3}\right]$$

$$= \frac{3\sqrt{3}}{2} \approx 2.598.$$



Error: $\pi - \dfrac{3\sqrt{3}}{2} \approx 0.5435.$

(b) There are n triangles, each with central angle of $\theta = 2\pi/n$. Hence,

$$An = n\left[\frac{1}{2}bh\right] = n\left[\frac{1}{2}(1)\sin\frac{2\pi}{n}\right] = \frac{n\sin(2\pi/n)}{2}.$$

(c)

n	6	12	24	48	96
An	2.598	3	3.106	3.133	3.139

(d) As n gets larger and larger, $2\pi/n$ approaches 0.

Letting $x = 2\pi/n$,

$$An = \frac{\sin(2\pi/n)}{2/n} = \frac{\sin(2\pi/n)}{(2\pi/n)}\pi = \frac{\sin x}{x}\pi$$

which approaches $(1)\pi = \pi$.

5. (a) Slope $= -\dfrac{12}{5}$

(b) Slope of tangent line is $\dfrac{5}{12}$.

$$y + 12 = \frac{5}{12}(x - 5)$$

$$y = \frac{5}{12}x - \frac{169}{12} \quad \text{Tangent line}$$

(c) $Q = (x, y) = \left(x, \sqrt{169 - x^2}\right)$

$$m_x = \frac{-\sqrt{169 - x^2} + 12}{x - 5}$$

(d) $$\lim_{x\to5} m_x = \lim_{x\to5} \frac{12 - \sqrt{169 - x^2}}{x - 5} \cdot \frac{12 + \sqrt{169 - x^2}}{12 + \sqrt{169 - x^2}}$$

$$= \lim_{x\to5} \frac{144 - (169 - x^2)}{(x - 5)\left(12 + \sqrt{169 - x^2}\right)}$$

$$= \lim_{x\to5} \frac{x^2 - 25}{(x - 5)\left(12 + \sqrt{169 - x^2}\right)}$$

$$= \lim_{x\to5} \frac{(x + 5)}{12 + \sqrt{169 - x^2}}$$

$$= \frac{10}{12 + 12} = \frac{5}{12}$$

This is the same slope as part (b).

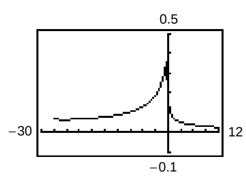
7. (a) $3 + x^{1/3} \ge 0$

$$x^{1/3} \ge -3$$

$$x \ge -27$$

Domain: $x \ge -27, x \ne 1$

(b)



(c) $$\lim_{x\to-27^+} f(x) = \frac{\sqrt{3 + (-27)^{1/3}} - 2}{-27 - 1}$$

$$= \frac{-2}{-28} = \frac{1}{14} \approx 0.0714$$

(d) $$\lim_{x\to1} f(x) = \lim_{x\to1} \frac{\sqrt{3 + x^{1/3}} - 2}{x - 1} \cdot \frac{\sqrt{3 + x^{1/3}} + 2}{\sqrt{3 + x^{1/3}} + 2}$$

$$= \lim_{x\to1} \frac{3 + x^{1/3} - 4}{(x - 1)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \lim_{x\to1} \frac{x^{1/3} - 1}{(x^{1/3} - 1)(x^{2/3} + x^{1/3} + 1)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \lim_{x\to1} \frac{1}{(x^{2/3} + x^{1/3} + 1)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

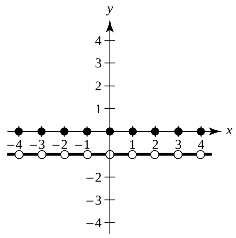
$$= \frac{1}{(1 + 1 + 1)(2 + 2)} = \frac{1}{12}$$

9. (a) $\lim_{x\to2} f(x) = 3$: g_1, g_4

(b) f continuous at 2: g_1

(c) $\lim_{x\to2^-} f(x) = 3$: g_1, g_3, g_4

11.



(a) $f(1) = [\![1]\!] + [\![-1]\!] = 1 + (-1) = 0$

$f(0) = 0$

$f\left(\frac{1}{2}\right) = 0 + (-1) = -1$

$f(-2.7) = -3 + 2 = -1$

(b) $\lim_{x\to 1^-} f(x) = -1$

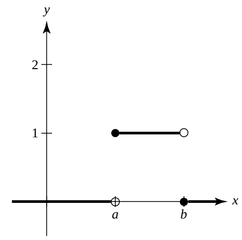
$\lim_{x\to 1^+} f(x) = -1$

$\lim_{x\to 1/2} f(x) = -1$

(c) f is continuous for all real numbers except

$x = 0, \pm 1, \pm 2, \pm 3, \ldots$

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) $P_{a,b}$ is continuous for all positive real numbers except $x = a, b$.

(d) The area under the graph of u, and above the x-axis, is 1.