

Review Exercises for Chapter 2

$$2. f(x) = \frac{x+1}{x-1}$$

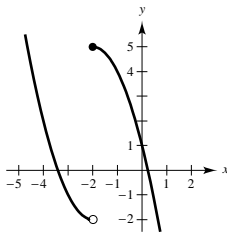
$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{x+\Delta x+1}{x+\Delta x-1} - \frac{x+1}{x-1}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x+1)(x-1) - (x+\Delta x-1)(x+1)}{\Delta x(x+\Delta x-1)(x-1)} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x^2 + x\Delta x + x - x - \Delta x - 1) - (x^2 + x\Delta x - x + x + \Delta x - 1)}{\Delta x(x+\Delta x-1)(x-1)} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-2\Delta x}{\Delta x(x+\Delta x-1)(x-1)} = \lim_{\Delta x \rightarrow 0} \frac{-2}{(x+\Delta x-1)(x-1)} = \frac{-2}{(x-1)^2} \end{aligned}$$

$$4. f(x) = \frac{2}{x}$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{2}{x+\Delta x} - \frac{2}{x}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x - (2x + 2\Delta x)}{\Delta x(x+\Delta x)x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-2\Delta x}{\Delta x(x+\Delta x)x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-2}{(x+\Delta x)x} = \frac{-2}{x^2} \end{aligned}$$

$$8. f(x) = \begin{cases} x^2 + 4x + 2, & \text{if } x < -2 \\ 1 - 4x - x^2, & \text{if } x \geq -2 \end{cases}$$

- (a) Nonremovable discontinuity at $x = -2$.
 (b) Not differentiable at $x = -2$ because the function is discontinuous there.



$$12. (a) \text{ Using the limit definition, } f'(x) = \frac{-2}{(x+1)^2}.$$

At $x = 0, f'(0) = -2$. The tangent line is

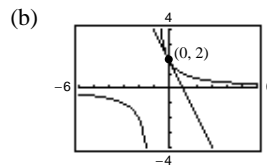
$$y - 2 = -2(x - 0)$$

$$y = -2x + 2$$

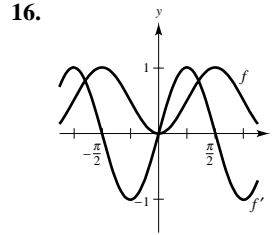
6. f is differentiable for all $x \neq -3$.

10. Using the limit definition, you obtain $h'(x) = \frac{3}{8} - 4x$.

$$\text{At } x = -2, h'(-2) = \frac{3}{8} - 4(-2) = \frac{67}{8}.$$



$$\begin{aligned}
 14. f'(2) &= \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} \\
 &= \lim_{x \rightarrow 2} \frac{\frac{1}{x+1} - \frac{1}{3}}{x - 2} \\
 &= \lim_{x \rightarrow 2} \frac{3 - x - 1}{(x - 2)(x + 1)3} \\
 &= \lim_{x \rightarrow 2} \frac{-1}{(x + 1)3} = \frac{-1}{9}
 \end{aligned}$$



$$\begin{aligned}
 18. y &= -12 \\
 y' &= 0
 \end{aligned}$$

$$\begin{aligned}
 20. g(x) &= x^{12} \\
 g'(x) &= 12x^{11}
 \end{aligned}$$

$$\begin{aligned}
 22. f(t) &= -8t^5 \\
 f'(t) &= -40t^4
 \end{aligned}$$

$$\begin{aligned}
 24. g(s) &= 4s^4 - 5s^2 \\
 g'(s) &= 16s^3 - 10s
 \end{aligned}$$

$$\begin{aligned}
 26. f(x) &= x^{1/2} - x^{-1/2} \\
 f'(x) &= \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{x+1}{2x^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 28. h(x) &= \frac{2}{9}x^{-2} \\
 h'(x) &= \frac{-4}{9}x^{-3} = \frac{-4}{9x^3}
 \end{aligned}$$

$$\begin{aligned}
 30. g(\alpha) &= 4 \cos \alpha + 6 \\
 g'(\alpha) &= -4 \sin \alpha
 \end{aligned}$$

$$\begin{aligned}
 32. g(\alpha) &= \frac{5 \sin \alpha}{3} - 2\alpha \\
 g'(\alpha) &= \frac{5 \cos \alpha}{3} - 2
 \end{aligned}$$

$$34. s = -16t^2 + s_0$$

First ball:

$$-16t^2 + 100 = 0$$

$$t = \sqrt{\frac{100}{16}} = \frac{10}{4} = 2.5 \text{ seconds to hit ground}$$

Second ball:

$$-16t^2 + 75 = 0$$

$$t^2 = \sqrt{\frac{75}{16}} = \frac{5\sqrt{3}}{4} \approx 2.165 \text{ seconds to hit ground}$$

Since the second ball was released one second after the first ball, the first ball will hit the ground first. The second ball will hit the ground $3.165 - 2.5 = 0.665$ second later.

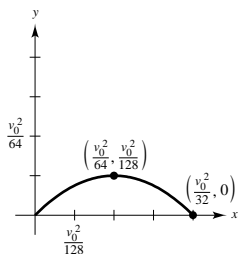
$$36. s(t) = -16t^2 + 14,400 = 0$$

$$16t^2 = 14,400$$

$$t = 30 \text{ sec}$$

Since $600 \text{ mph} = \frac{1}{6} \text{ mi/sec}$, in 30 seconds the bomb will move horizontally $(\frac{1}{6})(30) = 5$ miles.

38.



$$(a) \quad y = x - \frac{32}{v_0^2}x^2 = x\left(1 - \frac{32}{v_0^2}x\right)$$

$$= 0 \text{ if } x = 0 \text{ or } x = \frac{v_0^2}{32}.$$

Projectile strikes the ground when $x = v_0^2/32$.

Projectile reaches its maximum height at $x = v_0^2/64$.
(one-half the distance)

$$(c) \quad y = x - \frac{32}{v_0^2}x^2 = x\left(1 - \frac{32}{v_0^2}x\right) = 0$$

when $x = 0$ and $x = v_0^2/32$. Therefore, the range is $x = v_0^2/32$. When the initial velocity is doubled the range is

$$x = \frac{(2v_0)^2}{32} = \frac{4v_0^2}{32}$$

or four times the initial range. From part (a), the maximum height occurs when $x = v_0^2/64$. The maximum height is

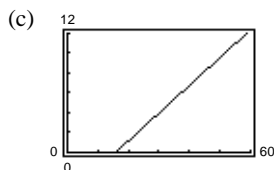
$$y\left(\frac{v_0^2}{64}\right) = \frac{v_0^2}{64} - \frac{32}{v_0^2}\left(\frac{v_0^2}{64}\right)^2 = \frac{v_0^2}{64} - \frac{v_0^2}{128} = \frac{v_0^2}{128}.$$

If the initial velocity is doubled, the maximum height is

$$y\left[\frac{(2v_0)^2}{64}\right] = \frac{(2v_0)^2}{128} = 4\left(\frac{v_0^2}{128}\right)$$

or four times the original maximum height.

$$40. (a) \quad y = 0.14x^2 - 4.43x + 58.4$$



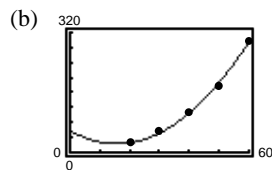
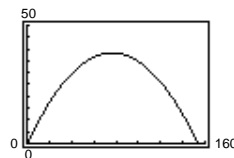
$$(b) \quad y' = 1 - \frac{64}{v_0^2}x$$

$$\text{When } x = \frac{v_0^2}{64}, y' = 1 - \frac{64}{v_0^2}\left(\frac{v_0^2}{64}\right) = 0.$$

$$(d) \quad v_0 = 70 \text{ ft/sec}$$

$$\text{Range: } x = \frac{v_0^2}{32} = \frac{(70)^2}{32} = 153.125 \text{ ft}$$

$$\text{Maximum height: } y = \frac{v_0^2}{128} = \frac{(70)^2}{128} \approx 38.28 \text{ ft}$$



$$(d) \quad \text{If } x = 65, y \approx 362 \text{ feet.}$$

(e) As the speed increases, the stopping distance increases at an increasing rate.

42. $g(x) = (x^3 - 3x)(x + 2)$

$$\begin{aligned} g'(x) &= (x^3 - 3x)(1) + (x + 2)(3x^2 - 3) \\ &= x^3 - 3x + 3x^3 + 6x^2 - 3x - 6 \\ &= 4x^3 + 6x^2 - 6x - 6 \end{aligned}$$

46. $f(x) = \frac{x+1}{x-1}$

$$\begin{aligned} f'(x) &= \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} \\ &= \frac{-2}{(x-1)^2} \end{aligned}$$

50. $f(x) = 9(3x^2 - 2x)^{-1}$

$$f'(x) = -9(3x^2 - 2x)^{-2}(6x - 2) = \frac{18(1 - 3x)}{(3x^2 - 2x)^2}$$

54. $y = 2x - x^2 \tan x$

$$y' = 2 - x^2 \sec^2 x - 2x \tan x$$

58. $v(t) = 36 - t^2, 0 \leq t \leq 6$

$$a(t) = v'(t) = -2t$$

$$v(4) = 36 - 16 = 20 \text{ m/sec}$$

$$a(4) = -8 \text{ m/sec}$$

62. $h(t) = 4 \sin t - 5 \cos t$

$$h'(t) = 4 \cos t + 5 \sin t$$

$$h''(t) = -4 \sin t + 5 \cos t$$

66. $f(x) = (x^2 - 1)^{1/3}$

$$\begin{aligned} f'(x) &= \frac{1}{3}(x^2 - 1)^{-2/3}(2x) \\ &= \frac{2x}{3(x^2 - 1)^{2/3}} \end{aligned}$$

44. $f(t) = t^3 \cos t$

$$\begin{aligned} f'(t) &= t^3(-\sin t) + \cos t(3t^2) \\ &= -t^3 \sin t + 3t^2 \cos t \end{aligned}$$

48. $f(x) = \frac{6x-5}{x^2+1}$

$$\begin{aligned} f'(x) &= \frac{(x^2+1)(6) - (6x-5)(2x)}{(x^2+1)^2} \\ &= \frac{2(3+5x-3x^2)}{(x^2+1)^2} \end{aligned}$$

52. $y = \frac{\sin x}{x^2}$

$$y' = \frac{(x^2) \cos x - (\sin x)(2x)}{x^4} = \frac{x \cos x - 2 \sin x}{x^3}$$

56. $y = \frac{1 + \sin x}{1 - \sin x}$

$$\begin{aligned} y' &= \frac{(1 - \sin x) \cos x - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2} \\ &= \frac{2 \cos x}{(1 - \sin x)^2} \end{aligned}$$

60. $f(x) = 12x^{1/4}$

$$f'(x) = 3x^{-3/4}$$

$$f''(x) = \frac{-9}{4}x^{-7/4} = \frac{-9}{4x^{7/4}}$$

64. $y = \frac{(10 - \cos x)}{x}$

$$xy + \cos x = 10$$

$$xy' + y - \sin x = 0$$

$$xy' = \sin x - y$$

$$xy' + y = (\sin x - y) + y = \sin x$$

68. $f(x) = \left(x^2 + \frac{1}{x}\right)^5$

$$f'(x) = 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

$$70. \quad h(\theta) = \frac{\theta}{(1-\theta)^3}$$

$$h'(\theta) = \frac{(1-\theta)^3 - \theta[3(1-\theta)^2(-1)]}{(1-\theta)^6}$$

$$= \frac{(1-\theta)^2(1-\theta+3\theta)}{(1-\theta)^6} = \frac{2\theta+1}{(1-\theta)^4}$$

$$74. \quad y = \csc 3x + \cot 3x$$

$$y' = -3 \csc 3x \cot 3x - 3 \csc^2 3x$$

$$= -3 \csc 3x(\cot 3x + \csc 3x)$$

$$78. \quad f(x) = \frac{3x}{\sqrt{x^2+1}}$$

$$f'(x) = \frac{3(x^2+1)^{1/2} - 3x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$$

$$= \frac{3(x^2+1) - 3x^2}{(x^2+1)^{3/2}}$$

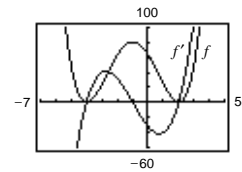
$$= \frac{3}{(x^2+1)^{3/2}}$$

$$82. \quad f(x) = [(x-2)(x+4)]^2 = (x^2+2x-8)^2$$

$$f'(x) = 4(x^3+3x^2-6x-8)$$

$$= 4(x-2)(x+1)(x+4)$$

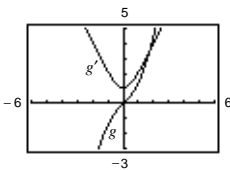
The zeros of f' correspond to the points on the graph of f where the tangent line is horizontal.



$$84. \quad g(x) = x(x^2+1)^{1/2}$$

$$g'(x) = \frac{2x^2+1}{\sqrt{x^2+1}}$$

g' does not equal zero for any value of x . The graph of g has no horizontal tangent lines.



$$72. \quad y = 1 - \cos 2x + 2 \cos^2 x$$

$$y' = 2 \sin 2x - 4 \cos x \sin x$$

$$= 2[2 \sin x \cos x] - 4 \sin x \cos x$$

$$= 0$$

$$76. \quad y = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5}$$

$$y' = \sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x)$$

$$= \sec^5 x \tan x(\sec^2 x - 1)$$

$$= \sec^5 x \tan^3 x$$

$$80. \quad y = \frac{\cos(x-1)}{x-1}$$

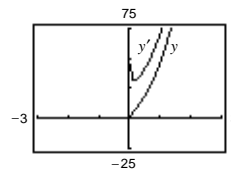
$$y' = \frac{-(x-1) \sin(x-1) - \cos(x-1)(1)}{(x-1)^2}$$

$$= -\frac{1}{(x-1)^2}[(x-1) \sin(x-1) + \cos(x-1)]$$

$$86. \quad y = \sqrt{3x}(x+2)^3$$

$$y' = \frac{3(x+2)^2(7x+2)}{2\sqrt{3x}}$$

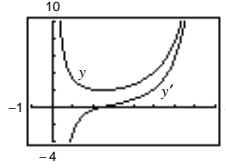
y' does not equal zero for any x in the domain. The graph has no horizontal tangent lines.



88. $y = 2 \csc^3(\sqrt{x})$

$$y' = -\frac{3}{\sqrt{x}} \csc^3 \sqrt{x} \cot \sqrt{x}$$

The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



90. $y = x^{-1} + \tan x$

$$y' = -x^{-2} + \sec^2 x$$

$$y'' = 2x^{-3} + 2 \sec x(\sec x \tan x)$$

$$= \frac{2}{x^3} + 2 \sec^2 x \tan x$$

96. $h(x) = x\sqrt{x^2 - 1}$

$$h'(x) = \frac{2x^2 - 1}{\sqrt{x^2 - 1}}$$

$$h''(x) = \frac{x(2x^2 - 3)}{(x^2 - 1)^{3/2}}$$

100. $x^2 + 9y^2 - 4x + 3y = 0$

$$2x + 18yy' - 4 + 3y' = 0$$

$$3(6y + 1)y' = 4 - 2x$$

$$y' = \frac{4 - 2x}{3(6y + 1)}$$

104. $\cos(x + y) = x$

$$-(1 + y') \sin(x + y) = 1$$

$$-y' \sin(x + y) = 1 + \sin(x + y)$$

$$y' = -\frac{1 + \sin(x + y)}{\sin(x + y)}$$

$$= -\csc(x + 1) - 1$$

92. $y = \sin^2 x$

$$y' = 2 \sin x \cos x = \sin 2x$$

$$y'' = 2 \cos 2x$$

94. $g(x) = \frac{6x - 5}{x^2 + 1}$

$$g'(x) = \frac{2(-3x^2 + 5x + 3)}{(x^2 + 1)^2}$$

$$g''(x) = \frac{2(6x^3 - 15x^2 - 18x + 5)}{(x^2 + 1)^3}$$

98. $v = \sqrt{2gh} = \sqrt{2(32)h} = 8\sqrt{h}$

$$\frac{dv}{dh} = \frac{4}{\sqrt{h}}$$

(a) When $h = 9$, $\frac{dv}{dh} = \frac{4}{3}$ ft/sec.

(b) When $h = 4$, $\frac{dv}{dh} = 2$ ft/sec.

102. $y^2 = x^3 - x^2y + xy - y^2$

$$0 = x^3 - x^2y + xy - 2y^2$$

$$0 = 3x^2 - x^2y' - 2xy + xy' + y - 4yy'$$

$$(x^2 - x + 4y)y' = 3x^2 - 2xy + y$$

$$y' = \frac{3x^2 - 2xy + y}{x^2 - x + 4y}$$

106. $x^2 - y^2 = 16$

$$2x - 2yy' = 0$$

$$y' = \frac{x}{y}$$

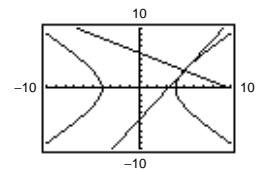
At (5, 3): $y' = \frac{5}{3}$

Tangent line: $y - 3 = \frac{5}{3}(x - 5)$

$$5x - 3y - 16 = 0$$

Normal line: $y - 3 = -\frac{3}{5}(x - 5)$

$$3x + 5y - 30 = 0$$



108. Surface area = $A = 6x^2$, x length of edge.

$$\frac{dx}{dt} = 5$$

$$\frac{da}{dt} = 12x \frac{dx}{dt} = 12(4.5)(5) = 270 \text{ cm}^2/\text{sec}$$

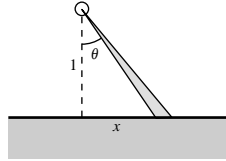
110. $\tan \theta = x$

$$\frac{d\theta}{dt} = 3(2\pi) \text{ rad/min}$$

$$\sec^2 \theta \left(\frac{d\theta}{dt} \right) = \frac{dx}{dt}$$

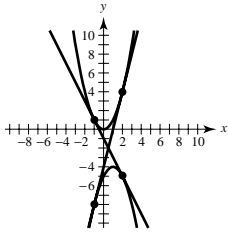
$$\frac{dx}{dt} = (\tan^2 \theta + 1)(6\pi) = 6\pi(x^2 + 1)$$

When $x = \frac{1}{2}$, $\frac{dx}{dt} = 6\pi \left(\frac{1}{4} + 1 \right) = \frac{15\pi}{2} \text{ km/min} = 450\pi \text{ km/hr.}$



Problem Solving for Chapter 2

2.



Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency.

For $y = x^2$, $y' = 2x$ and for $y = -x^2 + 2x - 5$, $y' = -2x + 2$.

Thus, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is

$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$\Rightarrow 2b^2 - 2b - 4 = 0$$

$$\Rightarrow b^2 - b - 2 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0$$

$$b = 2, -1$$

For $b = 2$, $a = 1 - b = -1$ and the points of tangency are $(-1, 1)$, $(2, -5)$. The tangent line has slope -2 :

$$y - 1 = -2(x - 1) \Rightarrow y = -2x - 1$$

For $b = -1$, $a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope 4 :

$$y - 4 = 4(x - 2) \Rightarrow y = 4x - 4.$$

4. (a) $y = x^2$, $y' = 2x$. Slope = 4 at $(2, 4)$.

Tangent line: $y - 4 = 4(x - 2)$

$$y = 4x - 4$$

(b) Slope of normal line: $-\frac{1}{4}$.

Normal line: $y - 4 = -\frac{1}{4}(x - 2)$

$$y = -\frac{1}{4}x + \frac{9}{2}$$

$$y = -\frac{1}{4}x + \frac{9}{2} = x^2 \Rightarrow 4x^2 + x - 18 = 0 \Rightarrow (4x + 9)(x - 2) = 0$$

$$x = 2, -\frac{9}{4}. \text{ Second intersection point: } \left(-\frac{9}{4}, \frac{81}{16}\right)$$

(c) Tangent line: $y = 0$

Normal line: $x = 0$

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4. —CONTINUED—

(d) Let (a, a^2) , $a \neq 0$, be a point on the parabola $y = x^2$. Tangent line at (a, a^2) is $y = 2a(x - a) + a^2$. Normal line at (a, a^2) is $y = -\frac{1}{2a}(x - a) + a^2$. To find points of intersection, solve

$$x^2 = -\frac{1}{2a}(x - a) + a^2$$

$$x^2 + \frac{1}{2a}x = a^2 + \frac{1}{2}$$

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x + \frac{1}{4a} = \pm \left(a + \frac{1}{4a}\right)$$

$$x + \frac{1}{4a} = a + \frac{1}{4a} \Rightarrow x = a \text{ (Point of tangency)}$$

$$x + \frac{1}{4a} = -\left(a + \frac{1}{4a}\right) \Rightarrow x = -a - \frac{1}{2a} = -\frac{2a^2 + 1}{2a}$$

The normal line intersects a second time at $x = -\frac{2a^2 + 1}{2a}$.

6. $f(x) = a + b \cos cx$

$$f'(x) = -bc \sin cx$$

$$\text{At } (0, 1): a + b = 1 \quad \text{Equation 1}$$

$$\text{At } \left(\frac{\pi}{4}, \frac{3}{2}\right): a + b \cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \quad \text{Equation 2}$$

$$-bc \sin\left(\frac{c\pi}{4}\right) = 1 \quad \text{Equation 3}$$

From Equation 1, $a = 1 - b$. Equation 2 becomes $(1 - b) + b \cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \Rightarrow -b + b \cos\frac{c\pi}{4} = \frac{1}{2}$

From Equation 3, $b = \frac{-1}{c \sin\left(\frac{c\pi}{4}\right)}$. Thus $\frac{1}{c \sin\left(\frac{c\pi}{4}\right)} + \frac{-1}{c \sin\left(\frac{c\pi}{4}\right)} \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}$

$$1 - \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2} c \sin\left(\frac{c\pi}{4}\right)$$

Graphing the equation $g(c) = \frac{1}{2} c \sin\left(\frac{c\pi}{4}\right) + \cos\left(\frac{c\pi}{4}\right) - 1$, you see that many values of c will work.

One answer: $c = 2$, $b = -\frac{1}{2}$, $a = \frac{3}{2} \Rightarrow f(x) = \frac{3}{2} - \frac{1}{2} \cos 2x$

8. (a) $b^2y^2 = x^3(a - x)$; $a, b > 0$

$$y^2 = \frac{x^3(a - x)}{b^2}$$

Graph $y_1 = \frac{\sqrt{x^3(a - x)}}{b}$ and $y_2 = -\frac{\sqrt{x^3(a - x)}}{b}$

(c) Differentiating implicitly.

$$2b^2yy' = 3x^2(a - x) - x^3 = 3ax^2 - 4x^3$$

$$y' = \frac{(3ax^2 - 4x^3)}{2b^2y} = 0$$

$$\Rightarrow 3ax^2 = 4x^3$$

$$3a = 4x$$

$$x = \frac{3a}{4}$$

$$b^2y^2 = \left(\frac{3a}{4}\right)^3 \left(a - \frac{3a}{4}\right) = \frac{27a^3}{64} \left(\frac{1}{4}a\right)$$

$$y^2 = \frac{27a^4}{256b^2} \Rightarrow y = \pm \frac{3\sqrt{3}a^2}{16b}$$

Two points: $\left(\frac{3a}{4}, \frac{3\sqrt{3}a^2}{16b}\right), \left(\frac{3a}{4}, -\frac{3\sqrt{3}a^2}{16b}\right)$

(b) a determines the x -intercept on the right: $(a, 0)$.

b affects the height.

10. (a) $y = x^{1/3} \Rightarrow \frac{dy}{dt} = \frac{1}{3}x^{-2/3} \frac{dx}{dt}$

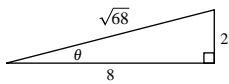
$$1 = \frac{1}{3}(8)^{-2/3} \frac{dx}{dt}$$

$$\frac{dx}{dt} = 12 \text{ cm/sec}$$

(b) $D = \sqrt{x^2 + y^2} \Rightarrow \frac{dD}{dt} = \frac{1}{2}(x^2 + y^2) \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt}\right)$

$$\begin{aligned} &= \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{\sqrt{x^2 + y^2}} \\ &= \frac{8(12) + 2(1)}{\sqrt{64 + 4}} = \frac{98}{\sqrt{68}} = \frac{49}{\sqrt{17}} \text{ cm/sec.} \end{aligned}$$

(c) $\tan \theta = \frac{y}{x} \Rightarrow \sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$



From the triangle, $\sec \theta = \frac{\sqrt{68}}{8}$. Hence $\frac{d\theta}{dt} = \frac{8(1) - 2(12)}{64 \left(\frac{68}{64}\right)} = \frac{-16}{68} = \frac{-4}{17} \text{ rad/sec}$

$$\begin{aligned} 12. E'(x) &= \lim_{\Delta x \rightarrow 0} \frac{E(x + \Delta x) - E(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{E(x)E(\Delta x) - E(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} E(x) \left(\frac{E(\Delta x) - 1}{\Delta x} \right) \\ &= E(x) \lim_{\Delta x \rightarrow 0} \frac{E(\Delta x) - 1}{\Delta x} \end{aligned}$$

$$\text{But, } E'(0) = \lim_{\Delta x \rightarrow 0} \frac{E(\Delta x) - E(0)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{E(\Delta x) - 1}{\Delta x} = 1.$$

Thus, $E'(x) = E(x)E'(0) = E(x)$ exists for all x .

For example: $E(x) = e^x$.

$$14. (a) v(t) = -\frac{27}{5}t + 27 \text{ ft/sec}$$

$$a(t) = -\frac{27}{5} \text{ ft/sec}^2$$

$$(b) v(t) = -\frac{27}{5}t + 27 = 0 \Rightarrow \frac{27}{5}t = 27 \Rightarrow t = 5 \text{ seconds}$$

$$S(5) = -\frac{27}{10}(5)^2 + 27(5) + 6 = 73.5 \text{ feet}$$

(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.