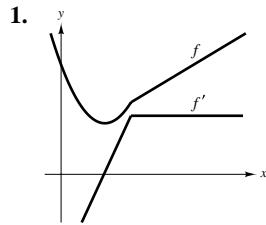


Review Exercises for Chapter 4



$$3. \int (2x^2 + x - 1) dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$$

$$5. \int \frac{x^3 + 1}{x^2} dx = \int \left(x + \frac{1}{x^2}\right) dx = \frac{1}{2}x^2 - \frac{1}{x} + C$$

$$7. \int (4x - 3 \sin x) dx = 2x^2 + 3 \cos x + C$$

$$9. f'(x) = -2x, (-1, 1)$$

$$11. a(t) = a$$

$$f(x) = \int -2x dx = -x^2 + C$$

$$v(t) = \int a dt = at + C_1$$

When $x = -1$:

$$v(0) = 0 + C_1 = 0 \text{ when } C_1 = 0.$$

$$y = -1 + C = 1$$

$$v(t) = at$$

$$C = 2$$

$$s(t) = \int at dt = \frac{a}{2}t^2 + C_2$$

$$y = 2 - x^2$$

$$s(0) = 0 + C_2 = 0 \text{ when } C_2 = 0.$$

$$s(t) = \frac{a}{2}t^2$$

$$s(30) = \frac{a}{2}(30)^2 = 3600 \text{ or}$$

$$a = \frac{2(3600)}{(30)^2} = 8 \text{ ft/sec}^2.$$

$$v(30) = 8(30) = 240 \text{ ft/sec}$$

$$13. a(t) = -32$$

$$15. (a) \sum_{i=1}^{10} (2i - 1)$$

$$v(t) = -32t + 96$$

$$s(t) = -16t^2 + 96t$$

$$(b) \sum_{i=1}^n i^3$$

$$(a) v(t) = -32t + 96 = 0 \text{ when } t = 3 \text{ sec.}$$

$$(b) s(3) = -144 + 288 = 144 \text{ ft}$$

$$(c) \sum_{i=1}^{10} (4i + 2)$$

$$(c) v(t) = -32t + 96 = \frac{96}{2} \text{ when } t = \frac{3}{2} \text{ sec.}$$

$$(d) s\left(\frac{3}{2}\right) = -16\left(\frac{9}{4}\right) + 96\left(\frac{3}{2}\right) = 108 \text{ ft}$$

17. $y = \frac{10}{x^2 + 1}, \Delta x = \frac{1}{2}, n = 4$

$$S(n) = S(4) = \frac{1}{2} \left[\frac{10}{1} + \frac{10}{(1/2)^2 + 1} + \frac{10}{(1)^2 + 1} + \frac{10}{(3/2)^2 + 1} \right]$$

$$\approx 13.0385$$

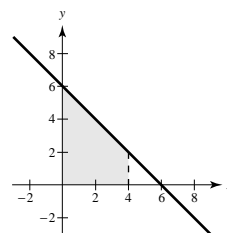
$$s(n) = s(4) = \frac{1}{2} \left[\frac{10}{(1/2)^2 + 1} + \frac{10}{1 + 1} + \frac{10}{(3/2)^2 + 1} + \frac{10}{2^2 + 1} \right]$$

$$\approx 9.0385$$

$$9.0385 < \text{Area of Region} < 13.0385$$

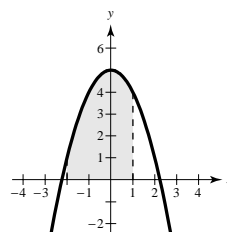
19. $y = 6 - x, \Delta x = \frac{4}{n},$ right endpoints

$$\begin{aligned} \text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(ci) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(6 - \frac{4i}{n} \right) \frac{4}{n} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left[6n - \frac{4n(n+1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} \left[24 - 8 \frac{n+1}{n} \right] = 24 - 8 = 16 \end{aligned}$$



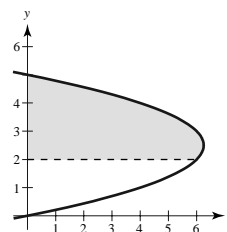
21. $y = 5 - x^2, \Delta x = \frac{3}{n}$

$$\begin{aligned} \text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(ci) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[5 - \left(-2 + \frac{3i}{n} \right)^2 \right] \left(\frac{3}{n} \right) \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[1 + \frac{12i}{n} - \frac{9i^2}{n^2} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \left[n + \frac{12n(n+1)}{2} - \frac{9n(n+1)(2n+1)}{6} \right] \\ &= \lim_{n \rightarrow \infty} \left[3 + 18 \frac{n+1}{n} - \frac{9(n+1)(2n+1)}{n^2} \right] \\ &= 3 + 18 - 9 = 12 \end{aligned}$$



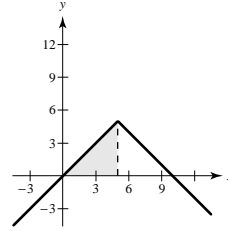
23. $x = 5y - y^2, 2 \leq y \leq 5, \Delta y = \frac{3}{n}$

$$\begin{aligned} \text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[5 \left(2 + \frac{3i}{n} \right) - \left(2 + \frac{3i}{n} \right)^2 \right] \left(\frac{3}{n} \right) \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[10 + \frac{15i}{n} - 4 - 12 \frac{i}{n} - \frac{9i^2}{n^2} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[6 + \frac{3i}{n} - \frac{9i^2}{n^2} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \left[6n + \frac{3n(n+1)}{2} - \frac{9n(n+1)(2n+1)}{6} \right] \\ &= \left[18 + \frac{9}{2} - 9 \right] = \frac{27}{2} \end{aligned}$$



$$25. \lim_{\|\Delta\| \rightarrow \infty} \sum_{i=1}^n (2ci - 3) \Delta xi = \int_4^6 (2x - 3) dx$$

27.



$$\int_0^5 (5 - |x - 5|) dx = \int_0^5 (5 - (5 - x)) dx = \int_0^5 x dx = \frac{25}{2}$$

(triangle)

$$29. (a) \int_2^6 [f(x) + g(x)] dx = \int_2^6 f(x) dx + \int_2^6 g(x) dx = 10 + 3 = 13$$

$$(b) \int_2^6 [f(x) - g(x)] dx = \int_2^6 f(x) dx - \int_2^6 g(x) dx = 10 - 3 = 7$$

$$(c) \int_2^6 [2f(x) - 3g(x)] dx = 2 \int_2^6 f(x) dx - 3 \int_2^6 g(x) dx = 2(10) - 3(3) = 11$$

$$(d) \int_2^6 5f(x) dx = 5 \int_2^6 f(x) dx = 5(10) = 50$$

$$31. \int_1^8 (\sqrt[3]{x} + 1) dx = \left[\frac{3}{4} x^{4/3} + x \right]_1^8 = \left[\frac{3}{4}(16) + 8 \right] - \left[\frac{3}{4} + 1 \right] = \frac{73}{4} \quad (c)$$

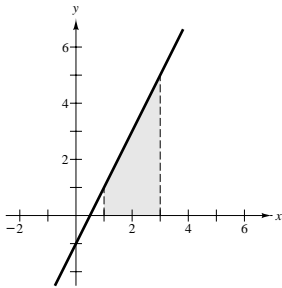
$$33. \int_0^4 (2 + x) dx = \left[2x + \frac{x^2}{2} \right]_0^4 = 8 + \frac{16}{2} = 16$$

$$35. \int_{-1}^1 (4t^3 - 2t) dt = \left[t^4 - t^2 \right]_{-1}^1 = 0$$

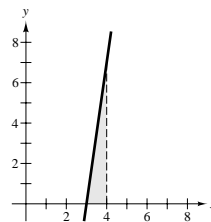
$$37. \int_4^9 x\sqrt{x} dx = \int_4^9 x^{3/2} dx = \left[\frac{2}{5} x^{5/2} \right]_4^9 = \frac{2}{5} [(\sqrt{9})^5 - (\sqrt{4})^5] = \frac{2}{5}(243 - 32) = \frac{422}{5}$$

$$39. \int_0^{3\pi/4} \sin \theta d\theta = \left[-\cos \theta \right]_0^{3\pi/4} = -\left(-\frac{\sqrt{2}}{2} \right) + 1 = 1 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + 2}{2}$$

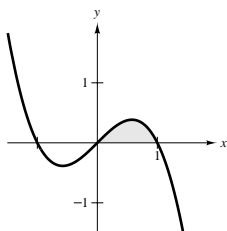
$$41. \int_1^3 (2x - 1) dx = \left[x^2 - x \right]_1^3 = 6$$



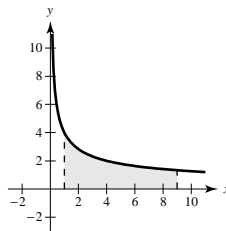
$$\begin{aligned} 43. \int_3^4 (x^2 - 9) dx &= \left[\frac{x^3}{3} - 9x \right]_3^4 \\ &= \left(\frac{64}{3} - 36 \right) - (9 - 27) \\ &= \frac{64}{3} - \frac{54}{3} = \frac{10}{3} \end{aligned}$$



$$45. \int_0^1 (x - x^3) dx = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$



$$47. \text{Area} = \int_1^9 \frac{4}{\sqrt{x}} dx = \left[\frac{4x^{1/2}}{(1/2)} \right]_1^9 = 8(3 - 1) = 16$$

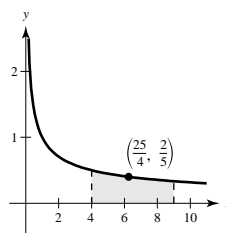


$$49. \frac{1}{9-4} \int_4^9 \frac{1}{\sqrt{x}} dx = \left[\frac{1}{5} 2\sqrt{x} \right]_4^9 = \frac{2}{5}(3 - 2) = \frac{2}{5} \text{ Average value}$$

$$\frac{2}{5} = \frac{1}{\sqrt{x}}$$

$$\sqrt{x} = \frac{5}{2}$$

$$x = \frac{25}{4}$$



$$51. F'(x) = x^2 \sqrt{1+x^3}$$

$$53. F'(x) = x^2 + 3x + 2$$

$$55. \int (x^2 + 1)^3 dx = \int (x^6 + 3x^4 + 3x^2 + 1) dx = \frac{x^7}{7} + \frac{3}{5}x^5 + x^3 + x + C$$

$$57. u = x^3 + 3, du = 3x^2 dx$$

$$\int \frac{x^2}{\sqrt{x^3 + 3}} dx = \int (x^3 + 3)^{-1/2} x^2 dx = \frac{1}{3} \int (x^3 + 3)^{-1/2} 3x^2 dx = \frac{2}{3}(x^3 + 3)^{1/2} + C$$

$$59. u = 1 - 3x^2, du = -6x dx$$

$$\int x(1 - 3x^2)^4 dx = -\frac{1}{6} \int (1 - 3x^2)^4 (-6x dx) = -\frac{1}{30}(1 - 3x^2)^5 + C = \frac{1}{30}(3x^2 - 1)^5 + C$$

$$61. \int \sin^3 x \cos x dx = \frac{1}{4} \sin^4 x + C$$

$$63. \int \frac{\sin \theta}{\sqrt{1 - \cos \theta}} dx = \int (1 - \cos \theta)^{-1/2} \sin \theta d\theta = 2(1 - \cos \theta)^{1/2} + C = 2\sqrt{1 - \cos \theta} + C$$

$$65. \int \tan^n x \sec^2 x dx = \frac{\tan^{n+1} x}{n+1} + C, n \neq -1$$

$$67. \int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x dx = \frac{1}{\pi} \int (1 + \sec \pi x)^2 (\pi \sec \pi x \tan \pi x) dx = \frac{1}{3\pi} (1 + \sec \pi x)^3 + C$$

$$69. \int_{-1}^2 x(x^2 - 4) dx = \frac{1}{2} \int_{-1}^2 (x^2 - 4)(2x) dx = \frac{1}{2} \left[\frac{(x^2 - 4)^2}{2} \right]_{-1}^2 = \frac{1}{4} [0 - 9] = -\frac{9}{4}$$

$$71. \int_0^3 \frac{1}{\sqrt{1+x}} dx = \int_0^3 (1+x)^{-1/2} dx = \left[2(1+x)^{1/2} \right]_0^3 = 4 - 2 = 2$$

$$73. u = 1 - y, y = 1 - u, dy = -du$$

When $y = 0$, $u = 1$. When $y = 1$, $u = 0$.

$$\begin{aligned} 2\pi \int_0^1 (y+1)\sqrt{1-y} dy &= 2\pi \int_1^0 -[(1-u)+1]\sqrt{u} du \\ &= 2\pi \int_1^0 (u^{3/2} - 2u^{1/2}) du = 2\pi \left[\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} \right]_1^0 = \frac{28\pi}{15} \end{aligned}$$

$$75. \int_0^\pi \cos\left(\frac{x}{2}\right) dx = 2 \int_0^\pi \cos\left(\frac{x}{2}\right) \frac{1}{2} dx = \left[2 \sin\left(\frac{x}{2}\right) \right]_0^\pi = 2$$

$$77. u = 1 - x, x = 1 - u, dx = -du$$

When $x = a$, $u = 1 - a$. When $x = b$, $u = 1 - b$.

$$\begin{aligned} P_{a,b} &= \int_a^b \frac{15}{4} x \sqrt{1-x} dx = \frac{15}{4} \int_{1-a}^{1-b} -(1-u)\sqrt{u} du \\ &= \frac{15}{4} \int_{1-a}^{1-b} (u^{3/2} - u^{1/2}) du = \frac{15}{4} \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \right]_{1-a}^{1-b} = \frac{15}{4} \left[\frac{2u^{3/2}}{15}(3u-5) \right]_{1-a}^{1-b} = \left[-\frac{(1-x)^{3/2}}{2}(3x+2) \right]_a^b \end{aligned}$$

$$(a) P_{0.50, 0.75} = \left[-\frac{(1-x)^{3/2}}{2}(3x+2) \right]_{0.50}^{0.75} = 0.353 = 35.3\%$$

$$(b) P_{0,b} = \left[-\frac{(1-x)^{3/2}}{2}(3x+2) \right]_0^b = -\frac{(1-b)^{3/2}}{2}(3b+2) + 1 = 0.5$$

$$(1-b)^{3/2}(3b+2) = 1$$

$$b \approx 0.586 = 58.6\%$$

$$79. p = 1.20 + 0.04t$$

$$C = \frac{15,000}{M} \int_t^{t+1} p ds$$

(a) 2000 corresponds to $t = 10$.

(b) 2005 corresponds to $t = 15$.

$$\begin{aligned} C &= \frac{15,000}{M} \int_{10}^{11} [1.20 + 0.04t] dt \\ &= \frac{15,000}{M} \left[1.20t + 0.02t^2 \right]_{10}^{11} = \frac{24,300}{M} \end{aligned}$$

$$C = \frac{15,000}{M} \left[1.20t + 0.02t^2 \right]_{15}^{16} = \frac{27,300}{M}$$

$$81. \text{Trapezoidal Rule } (n = 4): \int_1^2 \frac{1}{1+x^3} dx \approx \frac{1}{8} \left[\frac{1}{1+1^3} + \frac{2}{1+(1.25)^3} + \frac{2}{1+(1.5)^3} + \frac{2}{1+(1.75)^3} + \frac{1}{1+2^3} \right] \approx 0.257$$

$$\text{Simpson's Rule } (n = 4): \int_1^2 \frac{1}{1+x^3} dx \approx \frac{1}{12} \left[\frac{1}{1+1^3} + \frac{4}{1+(1.25)^3} + \frac{2}{1+(1.5)^3} + \frac{4}{1+(1.75)^3} + \frac{1}{1+2^3} \right] \approx 0.254$$

Graphing utility: 0.254

$$83. \text{Trapezoidal Rule } (n = 4): \int_0^{\pi/2} \sqrt{x} \cos x dx \approx 0.637$$

Simpson's Rule ($n = 4$): 0.685

Graphing Utility: 0.704

$$85. (a) R < I < T < L$$

$$(b) S(4) = \frac{4-0}{3(4)} [f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)]$$

$$\approx \frac{1}{3} \left[4 + 4(2) + 2(1) + 4\left(\frac{1}{2}\right) + \frac{1}{4} \right] \approx 5.417$$

Problem Solving for Chapter 4

1. (a) $L(1) = \int_1^1 \frac{1}{t} dt = 0$

(b) $L'(x) = \frac{1}{x}$ by the Second Fundamental Theorem of Calculus.

$$L'(1) = 1$$

(c) $L(x) = 1 = \int_1^x \frac{1}{t} dt$ for $x \approx 2.718$

$$\int_1^{2.718} \frac{1}{t} dt = 0.999896 \quad (\text{Note: The exact value of } x \text{ is } e, \text{ the base of the natural logarithm function.})$$

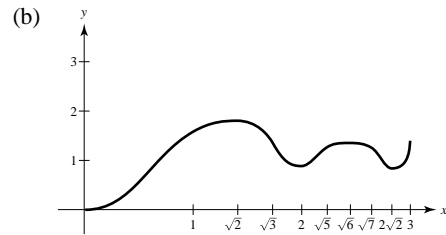
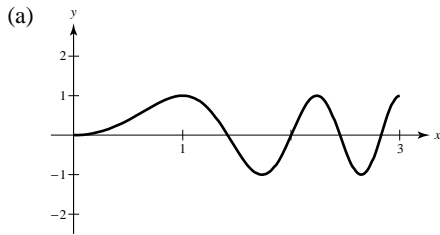
(d) We first show that $\int_1^{x_1} \frac{1}{t} dt = \int_{1/x_1}^1 \frac{1}{t} dt$.

To see this, let $u = \frac{t}{x_1}$ and $du = \frac{1}{x_1} dt$.

Then $\int_1^{x_1} \frac{1}{t} dt = \int_{1/x_1}^1 \frac{1}{x_1 u} (x_1 du) = \int_{1/x_1}^1 \frac{1}{u} du = \int_{1/x_1}^1 \frac{1}{t} dt$.

Now,
$$\begin{aligned} L(x_1 x_2) &= \int_1^{x_1 x_2} \frac{1}{t} dt = \int_{1/x_1}^{x_2} \frac{1}{u} du \quad (\text{using } u = \frac{t}{x_1}) \\ &= \int_{1/x_1}^1 \frac{1}{u} du + \int_1^{x_2} \frac{1}{u} du \\ &= \int_1^{x_1} \frac{1}{u} du + \int_1^{x_2} \frac{1}{u} du \\ &= L(x_1) + L(x_2). \end{aligned}$$

3. $S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt$



The zeros of $y = \sin \frac{\pi x^2}{2}$ correspond to the relative extrema of $S(x)$.

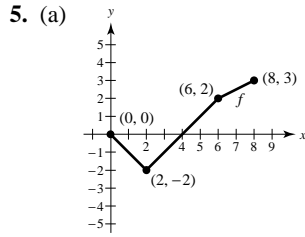
(c) $S'(x) = \sin \frac{\pi x^2}{2} = 0 \Rightarrow \frac{\pi x^2}{2} = n\pi \Rightarrow x^2 = 2n \Rightarrow x = \sqrt{2n}, n \text{ integer.}$

Relative maximum at $x = \sqrt{2} \approx 1.4142$ and $x = \sqrt{6} \approx 2.4495$

Relative minimum at $x = 2$ and $x = \sqrt{8} \approx 2.8284$

(d) $S''(x) = \cos\left(\frac{\pi x^2}{2}\right)(\pi x) = 0 \Rightarrow \frac{\pi x^2}{2} = \frac{\pi}{2} + n\pi \Rightarrow x^2 = 1 + 2n \Rightarrow x = \sqrt{1 + 2n}, n \text{ integer}$

Points of inflection at $x = 1, \sqrt{3}, \sqrt{5}, \text{ and } \sqrt{7}$.



(b)

x	0	1	2	3	4	5	6	7	8
$F(x)$	0	$-\frac{1}{2}$	-2	$-\frac{7}{2}$	-4	$-\frac{7}{2}$	-2	$\frac{1}{4}$	3

(c) $f(x) = \begin{cases} -x, & 0 \leq x < 2 \\ x - 4, & 2 \leq x < 6 \\ \frac{1}{2}x - 1, & 6 \leq x \leq 8 \end{cases}$

(d) $F''(x) = f'(x) = \begin{cases} -1, & 0 < x < 2 \\ 1, & 2 < x < 6 \\ \frac{1}{2}, & 6 < x < 8 \end{cases}$

$$F(x) = \int_0^x f(t) dt = \begin{cases} (-x^2/2), & 0 \leq x < 2 \\ (x^2/2) - 4x + 4, & 2 \leq x < 6 \\ (1/4)x^2 - x - 5, & 6 \leq x \leq 8 \end{cases}$$

$x = 2$ is a point of inflection, whereas $x = 6$ is not. (f is not continuous at $x = 6$.)

$F'(x) = f(x)$. F is decreasing on $(0, 4)$ and increasing on $(4, 8)$. Therefore, the minimum is -4 at $x = 4$, and the maximum is 3 at $x = 8$.

7. (a) $\int_{-1}^1 \cos x dx \approx \cos\left(-\frac{1}{\sqrt{3}}\right) + \cos\left(\frac{1}{\sqrt{3}}\right) = 2 \cos\left(\frac{1}{\sqrt{3}}\right) \approx 1.6758$

$$\int_{-1}^1 \cos x dx = \sin x \Big|_{-1}^1 = 2 \sin(1) \approx 1.6829$$

Error: $|1.6829 - 1.6758| = 0.0071$

(b) $\int_{-1}^1 \frac{1}{1+x^2} dx \approx \frac{1}{1+(1/3)} + \frac{1}{1+(1/3)} = \frac{3}{2}$

(Note: exact answer is $\pi/2 \approx 1.5708$)

(c) Let $p(x) = ax^3 + bx^2 + cx + d$.

$$\int_{-1}^1 p(x) dx = \left[\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx \right]_{-1}^1 = \frac{2b}{3} + 2d$$

$$p\left(-\frac{1}{\sqrt{3}}\right) + p\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{b}{3} + d\right) + \left(\frac{b}{3} + d\right) = \frac{2b}{3} + 2d$$

9. Consider $F(x) = [f(x)]^2 \Rightarrow F'(x) = 2f(x)f'(x)$. Thus,

$$\begin{aligned} \int_a^b f(x)f'(x) dx &= \int_a^b \frac{1}{2}F'(x) dx \\ &= \left[\frac{1}{2}F(x) \right]_a^b \\ &= \frac{1}{2}[F(b) - F(a)] \\ &= \frac{1}{2}[f(b)^2 - f(a)^2] \end{aligned}$$

11. Consider $\int_0^1 x^5 dx = \frac{x^6}{6} \Big|_0^1 = \frac{1}{6}$.

The corresponding Riemann Sum using right endpoints is

$$\begin{aligned} S(n) &= \frac{1}{n} \left[\left(\frac{1}{n}\right)^5 + \left(\frac{2}{n}\right)^5 + \dots + \left(\frac{n}{n}\right)^5 \right] \\ &= \frac{1}{n^6} [1^5 + 2^5 + \dots + n^5] \end{aligned}$$

Thus, $\lim_{n \rightarrow \infty} S(n) = \lim_{n \rightarrow \infty} \frac{1^5 + 2^5 + \dots + n^5}{n^6} = \frac{1}{6}$.

13. By Theorem 4.8, $0 < f(x) \leq M \Rightarrow \int_a^b f(x) dx \leq \int_a^b M dx = M(b - a)$.

Similarly, $m \leq f(x) \Rightarrow m(b - a) = \int_a^b m dx \leq \int_a^b f(x) dx$.

Thus, $m(b - a) \leq \int_a^b f(x) dx \leq M(b - a)$. On the interval $[0, 1]$, $1 \leq \sqrt{1 + x^4} \leq \sqrt{2}$ and $b - a = 1$.

Thus, $1 \leq \int_0^1 \sqrt{1 + x^4} dx \leq \sqrt{2}$. (Note: $\int_0^1 \sqrt{1 + x^4} dx \approx 1.0894$)

15. Since $-|f(x)| \leq f(x) \leq |f(x)|$,

$$-\int_a^b |f(x)| dx \leq \int_a^b f(x) dx \leq \int_a^b |f(x)| dx \Rightarrow \left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx.$$

17. $\frac{1}{365} \int_0^{365} 100,000 \left[1 + \sin \frac{2\pi(t - 60)}{365} \right] dt = \frac{100,000}{365} \left[t - \frac{365}{2\pi} \cos \frac{2\pi(t - 60)}{365} \right]_0^{365} = 100,000$ lbs.