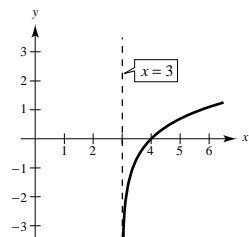


Review Exercises for Chapter 5

2. $f(x) = \ln(x - 3)$

Horizontal shift 3 units to the right

Vertical asymptote: $x = 3$ 

4. $\ln[(x^2 + 1)(x - 1)] = \ln(x^2 + 1) + \ln(x - 1)$

6. $3[\ln x - 2 \ln(x^2 + 1)] + 2 \ln 5 = 3 \ln x - 6 \ln(x^2 + 1) + \ln 5^2 = \ln x^3 - \ln(x^2 + 1)^6 + \ln 25 = \ln \left[\frac{25x^3}{(x^2 + 1)^6} \right]$

8. $\ln x + \ln(x - 3) = 0$

$$\ln x(x - 3) = 0$$

$$x(x - 3) = e^0$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$$x = \frac{3 + \sqrt{13}}{2} \text{ only since } \frac{3 - \sqrt{13}}{2} < 0.$$

10. $h(x) = \ln \frac{x(x-1)}{x-2} = \ln x + \ln(x-1) - \ln(x-2)$

$$h'(x) = \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x-2} = \frac{x^2 - 4x + 2}{x^3 - 3x^2 + 2x}$$

12. $f(x) = \ln[x(x^2 - 2)^{2/3}] = \ln x + \frac{2}{3} \ln(x^2 - 2)$

$$f'(x) = \frac{1}{x} + \frac{2}{3} \left(\frac{2x}{x^2 - 2} \right) = \frac{7x^2 - 6}{3x^3 - 6x}$$

14. $y = \frac{1}{b^2} [a + bx - a \ln(a + bx)]$

$$\frac{dy}{dx} = \frac{1}{b^2} \left(b - \frac{ab}{a + bx} \right) = \frac{x}{a + bx}$$

16. $y = -\frac{1}{ax} + \frac{b}{a^2} \ln \frac{a + bx}{x}$

$$= -\frac{1}{ax} + \frac{b}{a^2} [\ln(a + bx) - \ln x]$$

$$\frac{dy}{dx} = -\frac{1}{a} \left(-\frac{1}{x^2} \right) + \frac{b}{a^2} \left[\frac{b}{a + bx} - \frac{1}{x} \right]$$

$$= \frac{1}{ax^2} + \frac{b}{a^2} \left[\frac{-a}{x(a + bx)} \right] = \frac{1}{ax^2} - \frac{b}{ax(a + bx)}$$

$$= \frac{(a + bx) - bx}{ax^2(a + bx)} = \frac{1}{x^2(a + bx)}$$

18. $u = x^2 - 1, du = 2x dx$

$$\int \frac{x}{x^2 - 1} dx = \frac{1}{2} \int \frac{2x}{x^2 - 1} dx = \frac{1}{2} \ln|x^2 - 1| + C$$

20. $u = \ln x, du = \frac{1}{x} dx$

$$\int \frac{\ln \sqrt{x}}{x} dx = \frac{1}{2} \int (\ln x) \left(\frac{1}{x} \right) dx = \frac{1}{4} (\ln x)^2 + C$$

22. $\int_1^e \frac{\ln x}{x} dx = \int_1^e (\ln x)^1 \left(\frac{1}{x} \right) dx = \left[\frac{1}{2} (\ln x)^2 \right]_1^e = \frac{1}{2}$

24. $\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx = \left[\ln \left| \cos\left(\frac{\pi}{4} - x\right) \right| \right]_0^{\pi/4}$

$$= 0 - \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} \ln 2$$

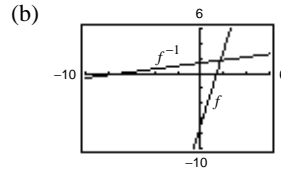
26. (a) $f(x) = 5x - 7$

$$y = 5x - 7$$

$$\frac{y + 7}{5} = x$$

$$\frac{x + 7}{5} = y$$

$$f^{-1}(x) = \frac{x + 7}{5}$$



(c) $f^{-1}(f(x)) = f^{-1}(5x - 7) = \frac{(5x - 7) + 7}{5} = x$

$$f(f^{-1}(x)) = f\left(\frac{x + 7}{5}\right) = 5\left(\frac{x + 7}{5}\right) - 7 = x$$

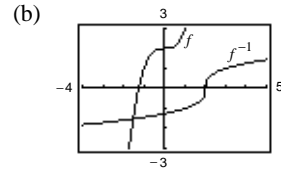
28. (a) $f(x) = x^3 + 2$

$$y = x^3 + 2$$

$$\sqrt[3]{y - 2} = x$$

$$\sqrt[3]{x - 2} = y$$

$$f^{-1}(x) = \sqrt[3]{x - 2}$$



(c) $f^{-1}(f(x)) = f^{-1}(x^3 + 2) = \sqrt[3]{(x^3 + 2) - 2} = x$

$$f(f^{-1}(x)) = f(\sqrt[3]{x - 2}) = (\sqrt[3]{x - 2})^3 + 2 = x$$

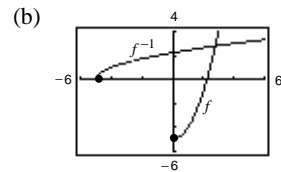
30. (a) $f(x) = x^2 - 5, x \geq 0$

$$y = x^2 - 5$$

$$\sqrt{y + 5} = x$$

$$\sqrt{x + 5} = y$$

$$f^{-1}(x) = \sqrt{x + 5}$$



(c) $f^{-1}(f(x)) = f^{-1}(x^2 - 5) = \sqrt{(x^2 - 5) + 5} = x$ for $x \geq 0$.

$$f(f^{-1}(x)) = f(\sqrt{x + 5}) = (\sqrt{x + 5})^2 - 5 = x$$

32. $f(x) = x\sqrt{x - 3}$

$$f(4) = 4$$

$$f'(x) = \sqrt{x - 3} + \frac{1}{2}x(x - 3)^{-1/2}$$

$$f'(4) = 1 + 2 = 3$$

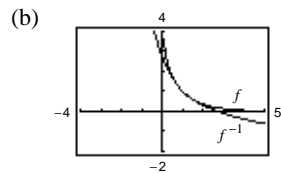
$$(f^{-1})'(4) = \frac{1}{f'(4)} = \frac{1}{3}$$

34. $f(x) = \ln x$

$$f^{-1}(x) = e^x$$

$$(f^{-1})'(x) = e^x$$

$$(f^{-1})'(0) = e^0 = 1$$



(c) $f^{-1}(f(x)) = f^{-1}(\ln(e^{1-x})) = 1 - \ln(e^{1-x})$

$$= 1 - (1 - x) = x$$

$$f(f^{-1}(x)) = f(1 - \ln x) = e^{1 - (1 - \ln x)} = e^{\ln x} = x$$

36. (a) $f(x) = e^{1-x}$

$$y = e^{1-x}$$

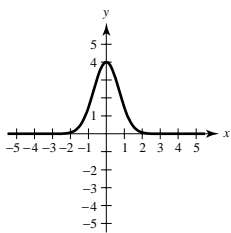
$$\ln y = 1 - x$$

$$x = 1 - \ln y$$

$$y = 1 - \ln x$$

$$f^{-1}(x) = 1 - \ln x$$

38. $y = 4e^{-x^2}$



40. $g(x) = \ln\left(\frac{e^x}{1+e^x}\right)$

$$= \ln e^x - \ln(1+e^x) = x - \ln(1+e^x)$$

$$g'(x) = 1 - \frac{e^x}{1+e^x} = \frac{1}{1+e^x}$$

42. $h(z) = e^{-z^2/2}$

$$h'(z) = -ze^{-z^2/2}$$

44. $y = 3e^{-3/t}$

$$y' = 3e^{-3/t}(3t^{-2}) = \frac{9e^{-3/t}}{t^2}$$

46. $f(\theta) = \frac{1}{2}e^{\sin 2\theta}$

$$f'(\theta) = \cos 2\theta e^{\sin 2\theta}$$

48. $\cos x^2 = xe^y$

$$-2x \sin x^2 = xe^y \frac{dy}{dx} + e^y$$

$$\frac{dy}{dx} = -\frac{2x \sin x^2 + e^y}{xe^y}$$

50. Let $u = \frac{1}{x}$, $du = -\frac{1}{x^2} dx$.

$$\int \frac{e^{1/x}}{x^2} dx = -\int e^{1/x} \left(-\frac{1}{x^2}\right) dx = -e^{1/x} + C$$

52. Let $u = e^{2x} + e^{-2x}$, $du = (2e^{2x} - e^{-2x}) dx$.

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx = \frac{1}{2} \int \frac{2e^{2x} - 2e^{-2x}}{e^{2x} + e^{-2x}} dx$$

$$= \frac{1}{2} \ln(e^{2x} + e^{-2x}) + C$$

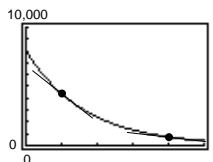
54. Let $u = x^3 + 1$, $du = 3x^2 dx$.

$$\int x^2 e^{x^3+1} dx = \frac{1}{3} \int e^{x^3+1} (3x^2) dx = \frac{1}{3} e^{x^3+1} + C$$

56. $\int \frac{e^{2x}}{e^{2x} + 1} dx = \frac{1}{2} \int \frac{1}{e^{2x} + 1} 2e^{2x} dx$

$$= \frac{1}{2} \ln(e^{2x} + 1) + C$$

58. (a), (c)



(b) $V = 8000e^{-0.6t}$, $0 \leq t \leq 5$

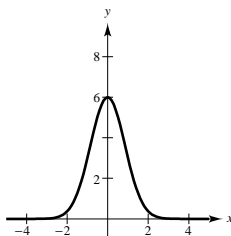
$$V'(t) = -4800e^{-0.6t}$$

$$V'(1) = -2634.3 \text{ dollars/year}$$

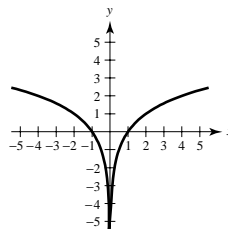
$$V'(4) = -435.4 \text{ dollars/year}$$

60. Area = $\int_0^2 2e^{-x} dx = \left[-2e^{-x}\right]_0^2 = -2e^{-2} + 2 = 2 - \frac{2}{e^2} \approx 1.729$

62. $g(x) = 6(2^{-x^2})$



64. $y = \log_4 x^2$



66. $f(x) = 4^x e^x$

$$f'(x) = 4^x e^x + (\ln 4)4^x e^x = 4^x e^x(1 + \ln 4)$$

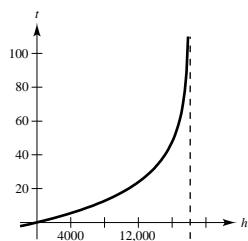
70. $h(x) = \log_5 \frac{x}{x-1} = \log_5 x - \log_5(x-1)$

$$h'(x) = \frac{1}{\ln 5} \left[\frac{1}{x} - \frac{1}{x-1} \right] = \frac{1}{\ln 5} \left[\frac{-1}{x(x-1)} \right]$$

74. $t = 50 \log_{10} \left(\frac{18,000}{18,000 - h} \right)$

(a) Domain: $0 \leq h < 18,000$

(b)

Vertical asymptote: $h = 18,000$

76. $2P = Pe^{10r}$

$$2 = e^{10r}$$

$$\ln 2 = 10r$$

$$r = \frac{\ln 2}{10} \approx 6.93\%$$

80. (a) $\frac{dy}{ds} = -0.012y, s > 50$

$$\frac{-1}{0.012} \int \frac{dy}{y} = \int ds$$

$$\frac{-1}{0.012} \ln y = s + C_1$$

$$y = Ce^{-0.012s}$$

When $s = 50, y = 28 = Ce^{-0.012(50)} \Rightarrow C = 28e^{0.6}$

$$y = 28e^{0.6 - 0.012s}, s > 50$$

82. $\frac{dy}{dx} = \frac{e^{-2x}}{1 + e^{-2x}}$

$$\int dy = \int \frac{e^{-2x}}{1 + e^{-2x}} dx = -\frac{1}{2} \int \frac{-2e^{-2x}}{1 + e^{-2x}} dx$$

$$y = -\frac{1}{2} \ln(1 + e^{-2x}) + C$$

68. $y = x(4^{-x})$

$$y' = 4^{-x} - x \cdot 4^{-x} \ln 4$$

72. $\int \frac{2^{-1/t}}{t^2} dt = \frac{1}{\ln 2} 2^{-1/t} + C$

(c) $t = 50 \log_{10} \left(\frac{18,000}{18,000 - h} \right)$

$$10^{t/50} = \frac{18,000}{18,000 - h}$$

$$18,000 - h = 18,000(10^{-t/50})$$

$$h = 18,000(1 - 10^{-t/50})$$

As $h \rightarrow 18,000, t \rightarrow \infty$.

(d) $t = 50 \log_{10} 18,000 - 50 \log_{10}(18,000 - h)$

$$\frac{dt}{dh} = \frac{50}{(\ln 10)(18,000 - h)}$$

$$\frac{d^2t}{dh^2} = \frac{50}{(\ln 10)(18,000 - h)^2}$$

No critical numbers

As t increases, the rate of change of the altitude is increasing.

78. $y = 5 \left(\frac{1}{2} \right)^{t/1620}$

$$y(600) = 5 \left(\frac{1}{2} \right)^{600/1620} \approx 3.868 \text{ grams}$$

(b)

Speed(s)	50	55	60	65	70
Miles per Gallon (y)	28	26.4	24.8	23.4	22.0

84. $y' - e^y \sin x = 0$

$$\frac{dy}{dx} = e^y \sin x$$

$$\int e^{-y} dy = \int \sin x dx$$

$$-e^{-y} = -\cos x + C_1$$

$$e^y = \frac{1}{\cos x + C} \quad (C = -C_1)$$

$$y = \ln \left| \frac{1}{\cos x + C} \right| = -\ln |\cos x + C|$$

86. $\frac{dy}{dx} = \frac{3(x+y)}{x}$ (homogeneous differential equation)

$$3(x+y) dx - x dy = 0$$

Let $y = vx$, $dy = x dv + v dx$.

$$3(x+vx) dx - x(x dv + v dx) = 0$$

$$(3x + 2vx) dx - x^2 dv = 0$$

$$(3 + 2v) dx = x dv$$

$$\int \frac{1}{x} dx = \int \frac{1}{3 + 2v} dv$$

$$\ln|x| = \frac{1}{2} \ln|3 + 2v| + C_1 = \ln(3 + 2v)^{1/2} + \ln C_2$$

$$x = C_2(3 + 2v)^{1/2}$$

$$x^2 = C(3 + 2v) = C\left(3 + 2\left(\frac{y}{x}\right)\right)$$

$$x^3 = C(3x + 2y) = 3Cx + 2Cy$$

$$y = \frac{x^3 - 3Cx}{2C}$$

88. $\frac{dv}{dt} = kv - 9.8$

(a) $\int \frac{dv}{kv - 9.8} = \int dt$

$$\frac{1}{k} \ln|kv - 9.8| = t + C_1$$

$$\ln|kv - 9.8| = kt + C_2$$

$$kv - 9.8 = e^{kt+C_2} = C_3 e^{kt}$$

$$v = \frac{1}{k} [9.8 + C_3 e^{kt}]$$

At $t = 0$, $v_0 = \frac{1}{k}(9.8 + C_3) \Rightarrow C_3 = kv_0 - 9.8$

$$v = \frac{1}{k} [9.8 + (kv_0 - 9.8)e^{kt}]$$

Note that $k < 0$ since the object is moving downward.

(b) $\lim_{t \rightarrow \infty} v(t) = \frac{9.8}{k}$

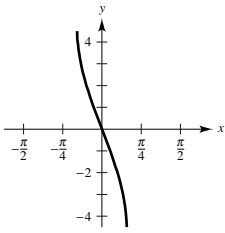
(c)
$$\begin{aligned} s(t) &= \int \frac{1}{k} [9.8 + (kv_0 - 9.8)e^{kt}] dt \\ &= \frac{1}{k} \left[9.8t + \frac{1}{k}(kv_0 - 9.8)e^{kt} \right] + C \\ &= \frac{9.8t}{k} + \frac{1}{k^2}(kv_0 - 9.8)e^{kt} + C \end{aligned}$$

$$s(0) = \frac{1}{k^2}(kv_0 - 9.8) + C \Rightarrow C = s_0 - \frac{1}{k^2}(kv_0 - 9.8)$$

$$s(t) = \frac{9.8t}{k} + \frac{1}{k^2}(kv_0 - 9.8)e^{kt} + s_0 - \frac{1}{k^2}(kv_0 - 9.8)$$

$$= \frac{9.8t}{k} + \frac{1}{k^2}(kv_0 - 9.8)(e^{kt} - 1) + s_0$$

90. $h(x) = -3 \arcsin(2x)$



94. $y = \arctan(x^2 - 1)$

$$y' = \frac{2x}{1 + (x^2 - 1)^2} = \frac{2x}{x^4 - 2x^2 + 2}$$

98. $y = \sqrt{x^2 - 4} - 2 \operatorname{arcsec} \frac{x}{2}, 2 < x < 4$

$$y' = \frac{x}{\sqrt{x^2 - 4}} - \frac{1}{(|x|/2)\sqrt{(x/2)^2 - 1}} = \frac{x}{\sqrt{x^2 - 4}} - \frac{4}{|x|\sqrt{x^2 - 4}} = \frac{x^2 - 4}{|x|\sqrt{x^2 - 4}} = \frac{\sqrt{x^2 - 4}}{x}$$

100. Let $u = 5x, du = 5 dx$.

$$\int \frac{1}{3 + 25x^2} dx = \frac{1}{5} \int \frac{1}{(\sqrt{3})^2 + (5x)^2} (5) dx = \frac{1}{5\sqrt{3}} \arctan \frac{5x}{\sqrt{3}} + C$$

102. $\int \frac{1}{16 + x^2} dx = \frac{1}{4} \arctan \frac{x}{4} + C$

104. $\int \frac{4-x}{\sqrt{4-x^2}} dx = 4 \int \frac{1}{\sqrt{4-x^2}} dx + \frac{1}{2} \int (4-x^2)^{-1/2} (-2x) dx = 4 \arcsin \frac{x}{2} + \sqrt{4-x^2} + C$

106. Let $u = \arcsin x, du = \frac{1}{\sqrt{1-x^2}} dx$.

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} dx = \frac{1}{2} (\arcsin x)^2 + C$$

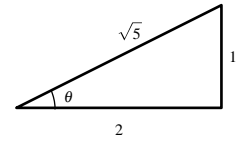
110. $y = x \tanh^{-1} 2x$

$$y' = x \left(\frac{2}{1-4x^2} \right) + \tanh^{-1} 2x = \frac{2x}{1-4x^2} + \tanh^{-1} 2x$$

92. (a) Let $\theta = \operatorname{arccot} 2$

$$\cot \theta = 2$$

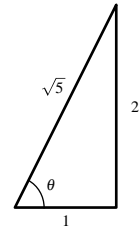
$$\tan(\operatorname{arccot} 2) = \tan \theta = \frac{1}{2}$$



(b) Let $\theta = \operatorname{arcsec} \sqrt{5}$

$$\sec \theta = \sqrt{5}$$

$$\cos(\operatorname{arcsec} \sqrt{5}) = \cos \theta = \frac{1}{\sqrt{5}}$$

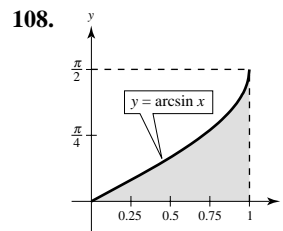


96. $y = \frac{1}{2} \arctan e^{2x}$

$$y' = \frac{1}{2} \left(\frac{1}{1 + e^{4x}} \right) (2e^{2x}) = \frac{e^{2x}}{1 + e^{4x}}$$

108. Let $u = \arcsin x, du = \frac{1}{\sqrt{1-x^2}} dx$.

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} dx = \frac{1}{2} (\arcsin x)^2 + C$$



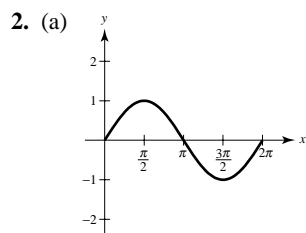
Since the area of region A is $1 \left(\int_0^{\pi/2} \sin y dy \right)$,

the shaded area is $\int_0^1 \arcsin x dx = \frac{\pi}{2} - 1 \approx 0.571$.

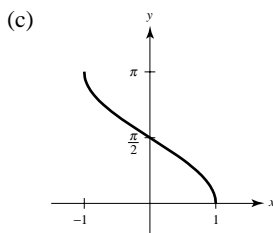
112. Let $u = x^3, du = 3x^2 dx$.

$$\int x^2 (\operatorname{sech} x^3)^2 dx = \frac{1}{3} \int (\operatorname{sech} x^3)^2 (3x^2) dx = \frac{1}{3} \tanh x^3 + C$$

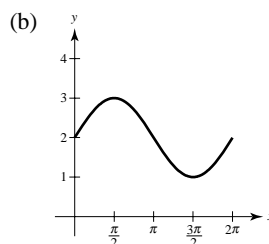
Problem Solving for Chapter 5



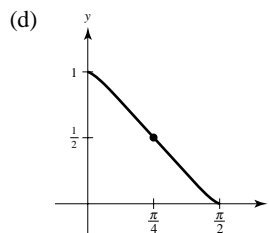
$$\int_0^{\pi} \sin x \, dx = -\int_{\pi}^{2\pi} \sin x \, dx \Rightarrow \int_0^{2\pi} \sin x \, dx = 0$$



$$\int_{-1}^1 \arccos x \, dx = 2\left(\frac{\pi}{2}\right) = \pi$$



$$\int_0^{2\pi} (\sin x + 2) \, dx = 2(2\pi) = 4\pi$$



$y = \frac{1}{1 + (\tan x)\sqrt{2}}$ is symmetric with respect to the point $\left(\frac{\pi}{4}, \frac{1}{2}\right)$. $\int_0^{\pi/2} \frac{1}{1 + (\tan x)\sqrt{2}} \, dx = \frac{\pi}{2} \left(\frac{1}{2}\right) = \frac{\pi}{4}$

4. $y = 0.5^x$ and $y = 1.2^x$ intersect $y = x$.

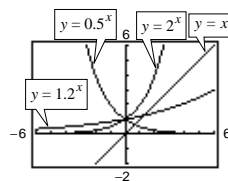
$y = 2^x$ does not intersect $y = x$.

Suppose $y = x$ is tangent to $y = a^x$ at (x, y) .

$$a^x = x \Rightarrow a = x^{1/x}$$

$$y' = a^x \ln a = 1 \Rightarrow x \ln x^{1/x} = 1 \Rightarrow \ln x = 1 \Rightarrow x = e, a = e^{1/e}$$

For $0 < a \leq e^{1/e} \approx 1.445$, the curve $y = a^x$ intersects $y = x$.



6. (a) $y = f(x) = \arcsin x$

$$\sin y = x$$

$$\text{Area } A = \int_{\pi/6}^{\pi/4} \sin y \cdot dy = -\cos y \Big|_{\pi/6}^{\pi/4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3} - \sqrt{2}}{2} \approx 0.1589$$

$$\text{Area } B = \left(\frac{1}{2}\right)\left(\frac{\pi}{6}\right) = \frac{\pi}{12} \approx 0.2618$$

(b) $\int_{1/2}^{\sqrt{2}/2} \arcsin x \, dx = \text{Area}(C) = \left(\frac{\pi}{4}\right)\left(\frac{\sqrt{2}}{2}\right) - A - B$

$$= \frac{\pi\sqrt{2}}{8} - \frac{\sqrt{3} - \sqrt{2}}{2} - \frac{\pi}{12}$$

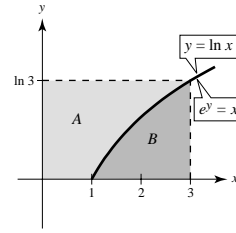
$$= \pi\left(\frac{\sqrt{2}}{8} - \frac{1}{12}\right) + \frac{\sqrt{2} - \sqrt{3}}{2} \approx 0.1346$$

—CONTINUED—

6. —CONTINUED—

$$\begin{aligned} \text{(c) Area } A &= \int_0^{\ln 3} e^y dy \\ &= e^y \Big|_0^{\ln 3} = 3 - 1 = 2 \end{aligned}$$

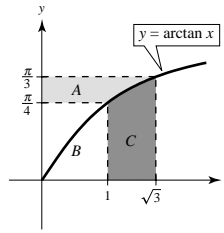
$$\text{Area } B = \int_1^3 \ln x dx = 3(\ln 3) - A = 3 \ln 3 - 2 = \ln 27 - 2 \approx 1.2958$$



$$\text{(d) } \tan y = x$$

$$\begin{aligned} \text{Area } A &= \int_{\pi/4}^{\pi/3} \tan y dy \\ &= -\ln|\cos y| \Big|_{\pi/4}^{\pi/3} \\ &= -\ln \frac{1}{2} + \ln \frac{\sqrt{2}}{2} = \ln \sqrt{2} = \frac{1}{2} \ln 2 \end{aligned}$$

$$\begin{aligned} \text{Area } C &= \int_1^{\sqrt{3}} \arctan x dx = \left(\frac{\pi}{3}\right)(\sqrt{3}) - \frac{1}{2} \ln 2 - \left(\frac{\pi}{4}\right)(1) \\ &= \frac{\pi}{12}(4\sqrt{3} - 3) - \frac{1}{2} \ln 2 \approx 0.6818 \end{aligned}$$



$$\begin{aligned} \text{8. } y &= e^x \\ y' &= e^x \\ y - b &= e^a(x - a) \\ y &= e^a x - ae^a + b \quad \text{Tangent line} \\ \text{If } y &= 0, \\ e^a x &= ae^a - b \\ bx &= ab - b \quad (b = e^a) \\ x &= a - 1 \\ c &= a - 1 \\ \text{Thus, } a - c &= a - (a - 1) = 1. \end{aligned}$$

$$\text{10. Let } u = \tan x, du = \sec^2 x dx$$

$$\begin{aligned} \text{Area} &= \int_0^{\pi/4} \frac{1}{\sin^2 x + 4 \cos^2 x} dx = \int_0^{\pi/4} \frac{\sec^2 x}{\tan^2 x + 4} dx \\ &= \int_0^1 \frac{du}{u^2 + 4} \\ &= \left[\frac{1}{2} \arctan\left(\frac{u}{2}\right) \right]_0^1 \\ &= \frac{1}{2} \arctan\left(\frac{1}{2}\right) \end{aligned}$$

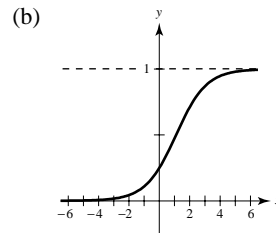
$$\text{12. (a) } \frac{dy}{dt} = y(1 - y), y(0) = \frac{1}{4}$$

$$\begin{aligned} \int \left(\frac{1}{y} + \frac{1}{1-y} \right) dy &= \int dt \\ \ln|y| - \ln|1-y| &= t + C \\ \ln \left| \frac{y}{1-y} \right| &= t + C \end{aligned}$$

$$\begin{aligned} \frac{y}{1-y} &= e^{t+C} = C_1 e^t \\ y &= C_1 e^t - y C_1 e^t \\ y &= \frac{C_1 e^t}{1 + C_1 e^t} = \frac{1}{1 + C_2 e^{-t}} \end{aligned}$$

$$y(0) = \frac{1}{4} = \frac{1}{1 + C_2} \Rightarrow C_2 = 3$$

$$\text{Hence, } y = \frac{1}{1 + 3e^{-t}}$$



$$\frac{dy}{dt} = y(1 - y) = y - y^2$$

$$\frac{d^2y}{dt^2} = y'' = y' - 2yy' \Rightarrow y'' = 0 \text{ for } y = \frac{1}{2}$$

$$\frac{d^2y}{dt^2} > 0 \text{ if } 0 < y < \frac{1}{2} \text{ and } \frac{d^2y}{dt^2} < 0 \text{ if } \frac{1}{2} < y < 1.$$

Thus, the rate of growth is maximum at $y = \frac{1}{2}$, the point of inflection.

—CONTINUED—

12. —CONTINUED—

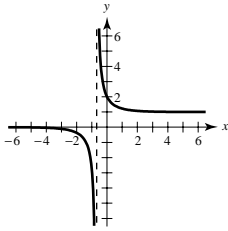
$$(c) \quad y' = y(1 - y), \quad y(0) = 2$$

$$\text{As before, } y = \frac{1}{1 + C_2 e^{-t}}$$

$$y(0) = 2 = \frac{1}{1 + C_2} \Rightarrow C_2 = -\frac{1}{2}$$

$$\text{Thus, } y = \frac{1}{1 - \frac{1}{2}e^{-t}} = \frac{2}{2 - e^{-t}}$$

The graph is different:



$$14. (a) \quad u = 985.93 - \left(985.93 - \frac{(120,000)(0.095)}{12}\right) \left(1 + \frac{0.095}{12}\right)^{12t}$$

$$v = \left(985.93 - \frac{(120,000)(0.095)}{12}\right) \left(1 + \frac{0.095}{12}\right)^{12t}$$

(b) The larger part goes for interest. The curves intersect when $t \approx 27.7$ years.

(c) The slopes are negatives of each other. Analytically,

$$u = 985.93 - v \Rightarrow \frac{du}{dt} = -\frac{dv}{dt}$$

$$u'(15) = -v'(15) = -14.06.$$

(d) $t = 12.7$ years

Again, the larger part goes for interest.

