

26. From Exercise 21:

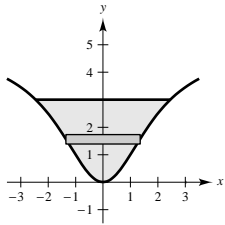
$$F = 64(15)\pi\left(\frac{1}{2}\right)^2 \approx 753.98 \text{ lb}$$

28. $h(y) = 3 - y$

Solving $y = 5x^2/(x^2 + 4)$ for x , you obtain
 $x = \sqrt{4y/(5 - y)}$.

$$L(y) = 2\sqrt{\frac{4y}{5 - y}}$$

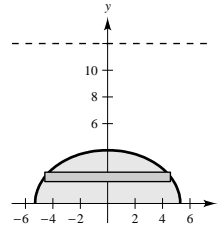
$$\begin{aligned} F &= 62.4(2) \int_0^3 (3 - y) \sqrt{\frac{4y}{5 - y}} dy \\ &= 2(124.8) \int_0^3 (3 - y) \sqrt{\frac{y}{5 - y}} dy \approx 546.265 \text{ lb} \end{aligned}$$



30. $h(y) = 12 - y$

$$L(y) = 2\frac{\sqrt{7(16 - y^2)}}{2} = \sqrt{7(16 - y^2)}$$

$$\begin{aligned} F &= 62.4 \int_0^4 (12 - y) \sqrt{7(16 - y^2)} dy \\ &= 62.4\sqrt{7} \int_0^4 (12 - y) \sqrt{16 - y^2} dy \approx 21373.7 \text{ lb} \end{aligned}$$

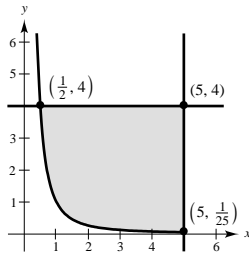


32. Fluid pressure is the force per unit of area exerted by a fluid over the surface of a body.

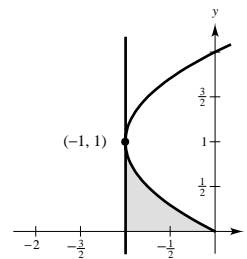
34. The left window experiences the greater fluid force because its centroid is lower.

Review Exercises for Chapter 6

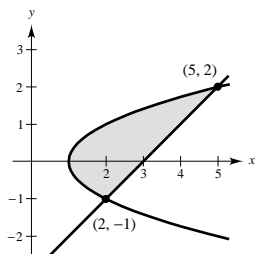
$$\begin{aligned} 2. A &= \int_{1/2}^5 \left(4 - \frac{1}{x^2}\right) dx \\ &= \left[4x + \frac{1}{x}\right]_{1/2}^5 = \frac{81}{5} \end{aligned}$$



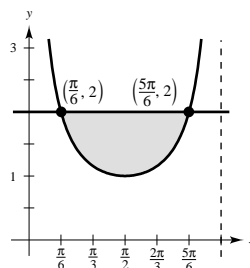
$$\begin{aligned} 4. A &= \int_0^1 [(y^2 - 2y) - (-1)] dy \\ &= \int_0^1 (y^2 - 2y + 1) dy \\ &= \int_0^1 (y - 1)^2 dy \\ &= \left[\frac{(y - 1)^3}{3}\right]_0^1 = \frac{1}{3} \end{aligned}$$



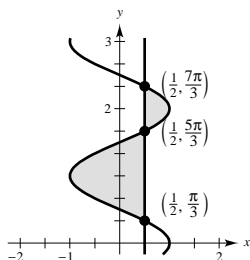
$$\begin{aligned}
 6. A &= \int_{-1}^2 [(y+3) - (y^2+1)] dy \\
 &= \int_{-1}^2 (2+y-y^2) dy \\
 &= \left[2y + \frac{1}{2}y^2 - \frac{1}{3}y^3 \right]_{-1}^2 = \frac{9}{2}
 \end{aligned}$$



$$\begin{aligned}
 8. A &= 2 \int_{\pi/6}^{\pi/2} (2 - \csc x) dx \\
 &= 2 \left[2x - \ln|\csc x - \cot x| \right]_{\pi/6}^{\pi/2} \\
 &= 2 \left([\pi - 0] - \left[\frac{\pi}{3} - \ln(2 - \sqrt{3}) \right] \right) \\
 &= 2 \left[\frac{2\pi}{3} + \ln(2 - \sqrt{3}) \right] \approx 1.555
 \end{aligned}$$



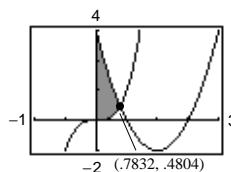
$$\begin{aligned}
 10. A &= \int_{\pi/3}^{5\pi/3} \left(\frac{1}{2} - \cos y \right) dy + \int_{5\pi/3}^{7\pi/3} \left(\cos y - \frac{1}{2} \right) dy \\
 &= \left[\frac{y}{2} - \sin y \right]_{\pi/3}^{5\pi/3} + \left[\sin y - \frac{y}{2} \right]_{5\pi/3}^{7\pi/3} \\
 &= \frac{\pi}{3} + 2\sqrt{3}
 \end{aligned}$$



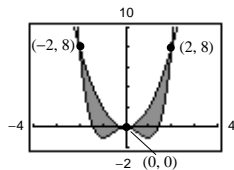
12. Point of intersection is given by:

$$x^3 - x^2 + 4x - 3 = 0 \Rightarrow x \approx 0.783.$$

$$\begin{aligned}
 A &\approx \int_0^{0.783} (3 - 4x + x^2 - x^3) dx \\
 &= \left[3x - 2x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 \right]_0^{0.783} \\
 &\approx 1.189
 \end{aligned}$$



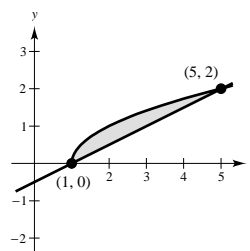
$$\begin{aligned}
 14. A &= 2 \int_0^2 [2x^2 - (x^4 - 2x^2)] dx \\
 &= 2 \int_0^2 (4x^2 - x^4) dx \\
 &= 2 \left[\frac{4}{3}x^3 - \frac{1}{5}x^5 \right]_0^2 = \frac{128}{15} \approx 8.5333
 \end{aligned}$$



$$16. y = \sqrt{x-1} \Rightarrow x = y^2 + 1$$

$$y = \frac{x-1}{2} \Rightarrow x = 2y + 1$$

$$\begin{aligned}
 A &= \int_0^2 [(2y+1) - (y^2+1)] dy \\
 &= \int_1^5 \left[\sqrt{x-1} - \frac{x-1}{2} \right] dx \\
 &= \left[\frac{2}{3}(x-1)^{3/2} - \frac{1}{4}(x-1)^2 \right]_1^5 = \frac{4}{3}
 \end{aligned}$$

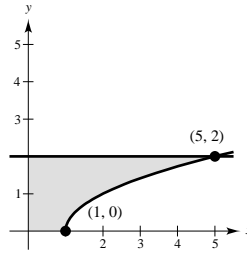


$$18. A = \int_0^1 2 \, dx + \int_1^5 [2 - \sqrt{x-1}] \, dx$$

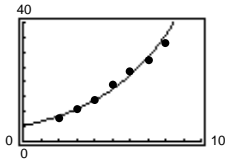
$$x = y^2 + 1$$

$$A = \int_0^2 (y^2 + 1) \, dy$$

$$= \left[\frac{1}{3}y^3 + y \right]_0^2 = \frac{14}{3}$$



$$20. (a) R_1(t) = 5.2834(1.2701)^t = 5.2834 e^{0.2391t}$$

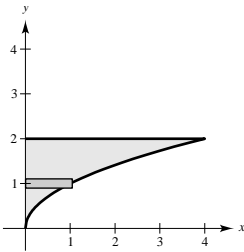


$$(b) R_2(t) = 10 + 5.28 e^{0.2t}$$

$$\text{Difference} = \int_{10}^{15} [R_1(t) - R_2(t)] \, dt \approx 171.25 \text{ billion dollars}$$

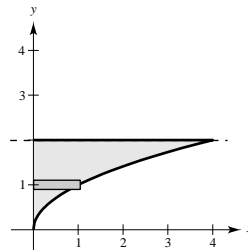
22. (a) Shell

$$V = 2\pi \int_0^2 y^3 \, dy = \left[\frac{\pi}{2} y^4 \right]_0^2 = 8\pi$$



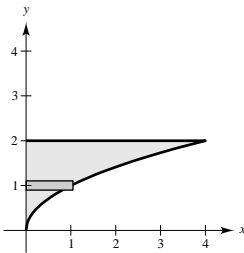
(b) Shell

$$\begin{aligned} V &= 2\pi \int_0^2 (2-y)y^2 \, dy \\ &= 2\pi \int_0^2 (2y^2 - y^3) \, dy \\ &= 2\pi \left[\frac{2}{3}y^3 - \frac{1}{4}y^4 \right]_0^2 = \frac{8\pi}{3} \end{aligned}$$



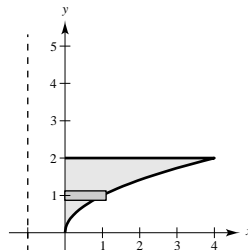
(c) Disk

$$V = \pi \int_0^2 y^4 \, dy = \left[\frac{\pi}{5} y^5 \right]_0^2 = \frac{32\pi}{5}$$



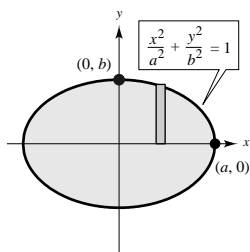
(d) Disk

$$\begin{aligned} V &= \pi \int_0^2 [(y^2 + 1)^2 - 1^2] \, dy \\ &= \pi \int_0^2 (y^4 + 2y^2) \, dy \\ &= \pi \left[\frac{1}{5}y^5 + \frac{2}{3}y^3 \right]_0^2 = \frac{176\pi}{15} \end{aligned}$$



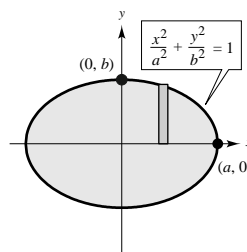
24. (a) Shell

$$\begin{aligned} V &= 4\pi \int_0^a (x) \frac{b}{a} \sqrt{a^2 - x^2} dx \\ &= \frac{-2\pi b}{a} \int_0^a (a^2 - x^2)^{1/2} (-2x) dx \\ &= \left[\frac{-4\pi b}{3a} (a^2 - x^2)^{3/2} \right]_0^a = \frac{4}{3} \pi a^2 b \end{aligned}$$



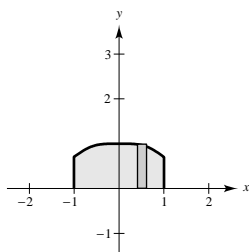
(b) Disk

$$\begin{aligned} V &= 2\pi \int_0^a \frac{b^2}{a^2} (a^2 - x^2) dx \\ &= \frac{2\pi b^2}{a^2} \left[a^2 x - \frac{1}{3} x^3 \right]_0^a \\ &= \frac{4}{3} \pi a b^2 \end{aligned}$$



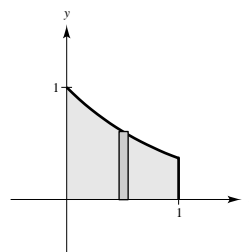
26. Disk

$$\begin{aligned} V &= 2\pi \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \right]^2 dx \\ &= \left[2\pi \arctan x \right]_0^1 \\ &= 2\pi \left(\frac{\pi}{4} - 0 \right) \\ &= \frac{\pi^2}{2} \end{aligned}$$



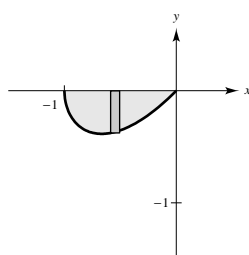
28. Disk

$$\begin{aligned} V &= \pi \int_0^1 (e^{-x})^2 dx \\ &= \pi \int_0^1 e^{-2x} dx = \left[-\frac{\pi}{2} e^{-2x} \right]_0^1 \\ &= \left(\frac{-\pi}{2e^2} + \frac{\pi}{2} \right) = \frac{\pi}{2} \left(1 - \frac{1}{e^2} \right) \end{aligned}$$



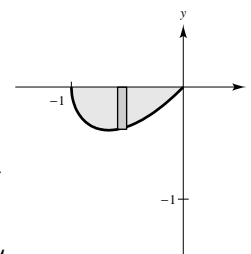
30. (a) Disk

$$\begin{aligned} V &= \pi \int_{-1}^0 x^2(x+1) dx \\ &= \pi \int_{-1}^0 (x^3 + x^2) dx \\ &= \pi \left[\frac{x^4}{4} + \frac{x^3}{3} \right]_{-1}^0 = \frac{\pi}{12} \end{aligned}$$



(b) Shell

$$\begin{aligned} u &= \sqrt{x+1} \\ x &= u^2 - 1 \\ dx &= 2u du \\ V &= 2\pi \int_{-1}^0 x^2 \sqrt{x+1} dx \\ &= 4\pi \int_0^1 (u^2 - 1)^2 u^2 du \\ &= 4\pi \int_0^1 (u^6 - 2u^4 + u^2) du \\ &= 4\pi \left[\frac{1}{7} u^7 - \frac{2}{5} u^5 + \frac{1}{3} u^3 \right]_0^1 = \frac{32\pi}{105} \end{aligned}$$



$$32. A(x) = \frac{1}{2}bh = \frac{1}{2}(2\sqrt{a^2 - x^2})(\sqrt{3}\sqrt{a^2 - x^2})$$

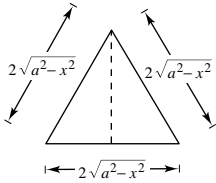
$$= \sqrt{3}(a^2 - x^2)$$

$$V = \sqrt{3} \int_{-a}^a (a^2 - x^2) dx = \sqrt{3} \left[a^2x - \frac{x^3}{3} \right]_{-a}^a$$

$$= \sqrt{3} \left(\frac{4a^3}{3} \right)$$

Since $(4\sqrt{3}a^3)/3 = 10$, we have $a^3 = (5\sqrt{3})/2$. Thus,

$$a = \sqrt[3]{\frac{5\sqrt{3}}{2}} \approx 1.630 \text{ meters.}$$



36. Since $f(x) = \tan x$ has $f'(x) = \sec^2 x$, this integral represents the length of the graph of $\tan x$ from $x = 0$ to $x = \pi/4$. This length is a little over 1 unit. Answers (b).

40. $F = kx$

$$50 = k(9) \Rightarrow k = \frac{50}{9}$$

$$F = \frac{50}{9}x$$

$$W = \int_0^9 \frac{50}{9}x dx = \left[\frac{25}{9}x^2 \right]_0^9$$

$$= 225 \text{ in} \cdot \text{lb} = 18.75 \text{ ft} \cdot \text{lb}$$

34. $y = \frac{x^3}{6} + \frac{1}{2x}$

$$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

$$1 + (y')^2 = \left(\frac{1}{2}x^2 + \frac{1}{2x^2} \right)^2$$

$$s = \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2x^2} \right) dx = \left[\frac{1}{6}x^3 - \frac{1}{2x} \right]_1^3 = \frac{14}{3}$$

38. $y = 2\sqrt{x}$

$$y' = \frac{1}{\sqrt{x}}$$

$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x+1}{x}$$

$$S = 2\pi \int_0^3 2\sqrt{x} \sqrt{\frac{x+1}{x}} dx = 4\pi \int_0^3 \sqrt{x+1} dx$$

$$= 4\pi \left[\frac{2}{3}(x+1)^{3/2} \right]_0^3 = \frac{56\pi}{3}$$

42. We know that

$$\frac{dV}{dt} = \frac{4 \text{ gal/min} - 12 \text{ gal/min}}{7.481 \text{ gal/ft}^3} = -\frac{8}{7.481} \text{ ft}^3/\text{min}$$

$$V = \pi r^2 h = \pi \left(\frac{1}{9} \right) h$$

$$\frac{dV}{dt} = \frac{\pi}{9} \left(\frac{dh}{dt} \right)$$

$$\frac{dh}{dt} = \frac{9}{\pi} \left(\frac{dV}{dt} \right) = \frac{9}{\pi} \left(-\frac{8}{7.481} \right) \approx -3.064 \text{ ft/min.}$$

Depth of water: $-3.064t + 150$

Time to drain well: $t = \frac{150}{3.064} \approx 49$ minutes

(49)(12) = 588 gallons pumped

Volume of water pumped in Exercise 41: 391.7 gallons

$$\frac{391.7}{52\pi} = \frac{588}{x\pi}$$

$$x = \frac{588(52)}{391.7} \approx 78$$

Work $\approx 78\pi$ ft \cdot ton

44. (a) Weight of section of cable:
- $4 \Delta x$

Distance: $200 - x$

$$W = 4 \int_0^{200} (200 - x) dx = \left[-2(200 - x)^2 \right]_0^{200} = 80,000 \text{ ft} \cdot \text{lb} = 40 \text{ ft} \cdot \text{ton}$$

- (b) Work to move 300 pounds 200 feet vertically:
- $200(300) = 60,000 \text{ ft} \cdot \text{lb} = 30 \text{ ft} \cdot \text{ton}$

Total work = work for drawing up the cable + work of lifting the load

$$= 40 \text{ ft} \cdot \text{ton} + 30 \text{ ft} \cdot \text{ton} = 70 \text{ ft} \cdot \text{ton}$$

46.
$$W = \int_a^b F(x) dx$$

$$F(x) = \begin{cases} -(2/9)x + 6, & 0 \leq x \leq 9 \\ -(4/3)x + 16, & 9 \leq x \leq 12 \end{cases}$$

$$\begin{aligned} W &= \int_0^9 \left(-\frac{2}{9}x + 6 \right) dx + \int_9^{12} \left(-\frac{4}{3}x + 16 \right) dx \\ &= \left[-\frac{1}{9}x^2 + 6x \right]_0^9 + \left[-\frac{2}{3}x^2 + 16x \right]_9^{12} \\ &= (-9 + 54) + (-96 + 192 + 54 - 144) = 51 \text{ ft} \cdot \text{lbs} \end{aligned}$$

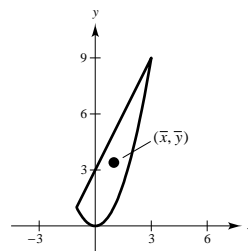
48.
$$A = \int_{-1}^3 [(2x + 3) - x^2] dx = \left[x^2 + 3x - \frac{1}{3}x^3 \right]_{-1}^3 = \frac{32}{3}$$

$$\frac{1}{A} = \frac{3}{32}$$

$$\bar{x} = \frac{3}{32} \int_{-1}^3 x(2x + 3 - x^2) dx = \frac{3}{32} \int_{-1}^3 (3x + 2x^2 - x^3) dx = \frac{3}{32} \left[\frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 \right]_{-1}^3 = 1$$

$$\begin{aligned} \bar{y} &= \left(\frac{3}{32} \right) \frac{1}{2} \int_{-1}^3 [(2x + 3)^2 - x^4] dx = \frac{3}{64} \int_{-1}^3 (9 + 12x + 4x^2 - x^4) dx \\ &= \frac{3}{64} \left[9x + 6x^2 + \frac{4}{3}x^3 - \frac{1}{5}x^5 \right]_{-1}^3 = \frac{17}{5} \end{aligned}$$

$$(\bar{x}, \bar{y}) = \left(1, \frac{17}{5} \right)$$



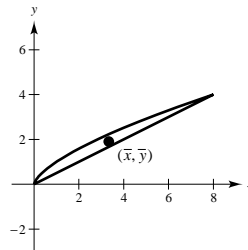
50.
$$A = \int_0^8 \left(x^{2/3} - \frac{1}{2}x \right) dx = \left[\frac{3}{5}x^{5/3} - \frac{1}{4}x^2 \right]_0^8 = \frac{16}{5}$$

$$\frac{1}{A} = \frac{5}{16}$$

$$\begin{aligned} \bar{x} &= \frac{5}{16} \int_0^8 x \left(x^{2/3} - \frac{1}{2}x \right) dx \\ &= \frac{5}{16} \left[\frac{3}{8}x^{8/3} - \frac{1}{6}x^3 \right]_0^8 = \frac{10}{3} \end{aligned}$$

$$\begin{aligned} \bar{y} &= \left(\frac{5}{16} \right) \frac{1}{2} \int_0^8 \left(x^{4/3} - \frac{1}{4}x^2 \right) dx \\ &= \frac{1}{2} \left(\frac{5}{16} \right) \left[\frac{3}{7}x^{7/3} - \frac{1}{12}x^3 \right]_0^8 = \frac{40}{21} \end{aligned}$$

$$(\bar{x}, \bar{y}) = \left(\frac{10}{3}, \frac{40}{21} \right)$$



52. Wall at shallow end:

$$F = 62.4 \int_0^5 y(20) dy = \left[(1248) \frac{y^2}{2} \right]_0^5 = 15,600 \text{ lb}$$

Wall at deep end:

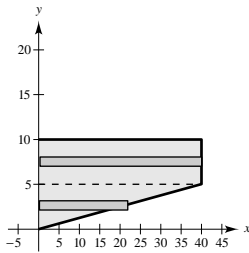
$$F = 62.4 \int_0^{10} y(20) dy = \left[(624)y^2 \right]_0^{10} = 62,400 \text{ lb}$$

Side wall:

$$F_1 = 62.4 \int_0^5 y(40) dy = \left[(1248)y^2 \right]_0^5 = 31,200 \text{ lb}$$

$$F_2 = 62.4 \int_0^5 (10 - y)8y dy = 62.4 \int_0^5 (80y - 8y^2) dy$$

$$F = F_1 + F_2 = 72,800 \text{ lb}$$



54. $F = 62.4(16\pi)5 = 4992\pi \text{ lb}$

Problem Solving for Chapter 6

2. $R = \int_0^1 x(1 - x) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Let (c, mc) be the intersection of the line and the parabola.

Then, $mc = c(1 - c) \Rightarrow m = 1 - c$ or $c = 1 - m$.

$$\frac{1}{2} \left(\frac{1}{6} \right) = \int_0^{1-m} (x - x^2 - mx) dx$$

$$\begin{aligned} \frac{1}{12} &= \left[\frac{x^2}{2} - \frac{x^3}{3} - m \frac{x^2}{2} \right]_0^{1-m} \\ &= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3} - m \frac{(1-m)^2}{2} \end{aligned}$$

$$1 = 6(1-m)^2 - 4(1-m)^3 - 6m(1-m)^2$$

$$= (1-m)^2(6 - 4(1-m) - 6m)$$

$$= (1-m)^2(2 - 2m)$$

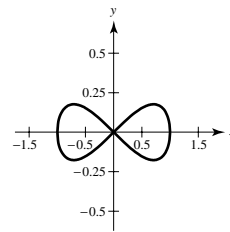
$$\frac{1}{2} = (1-m)^3$$

$$\left(\frac{1}{2} \right)^{1/3} = 1 - m$$

$$m = 1 - \left(\frac{1}{2} \right)^{1/3} \approx 0.2063$$

4. $8y^2 = x^2(1 - x^2)$

$$y = \pm \frac{|x|\sqrt{1-x^2}}{2\sqrt{2}}$$



For $x > 0$, $y' = \frac{1 - 2x^2}{2\sqrt{2}\sqrt{1-x^2}}$

$$\begin{aligned} S &= 2(2\pi) \int_0^1 x \sqrt{1 + \left(\frac{1 - 2x^2}{2\sqrt{2}\sqrt{1-x^2}} \right)^2} dx \\ &= \frac{5\sqrt{2}\pi}{3} \end{aligned}$$

6. By the Theorem of Pappus,

$$\begin{aligned} V &= 2\pi r A \\ &= 2\pi \left[d + \frac{1}{2} \sqrt{w^2 + l^2} \right] lw \end{aligned}$$

8. $f'(x)^2 = e^x$

$f'(x) = e^{x/2}$

$f(x) = 2e^{x/2} + C$

$f(0) = 0 \Rightarrow C = -2$

$f(x) = 2e^{x/2} - 2$

10. Let
- ρ_f
- be the density of the fluid and
- ρ_0
- the density of the iceberg. The buoyant force is

$$F = \rho_f g \int_{-h}^0 A(y) dy$$

where $A(y)$ is a typical cross section and g is the acceleration due to gravity. The weight of the object is

$$W = \rho_0 g \int_{-h}^{L-h} A(y) dy.$$

$F = W$

$$\rho_f g \int_{-h}^0 A(y) dy = \rho_0 g \int_{-h}^{L-h} A(y) dy$$

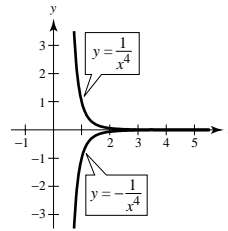
$$\frac{\rho_0}{\rho_f} = \frac{\text{submerged volume}}{\text{total volume}} = \frac{0.92 \times 10^3}{1.03 \times 10^3} = 0.893 \text{ or } 89.3\%$$

12. (a)
- $\bar{y} = 0$
- by symmetry

$$M_y = 2 \int_1^6 x \frac{1}{x^4} dx = 2 \int_1^6 \frac{1}{x^3} dx = \frac{35}{36}$$

$$m = 2 \int_1^6 \frac{1}{x^4} dx = \frac{215}{324}$$

$$\bar{x} = \frac{35/36}{215/324} = \frac{63}{43} \quad (\bar{x}, \bar{y}) = \left(\frac{63}{43}, 0 \right)$$



(b) $M_y = 2 \int_1^b \frac{1}{x^3} dx = \frac{b^2 - 1}{b^2}$

$$m = 2 \int_1^b \frac{1}{x^4} dx = \frac{2(b^3 - 1)}{3b^3}$$

$$\bar{x} = \frac{(b^2 - 1)/b^2}{2(b^3 - 1)/3b^3} = \frac{3b(b + 1)}{2(b^2 + b + 1)} \quad (\bar{x}, \bar{y}) = \left(\frac{3b(b + 1)}{2(b^2 + b + 1)}, 0 \right)$$

$$\lim_{b \rightarrow \infty} \bar{x} = \frac{3}{2} \quad (\bar{x}, \bar{y}) = \left(\frac{3}{2}, 0 \right)$$

14. (a) Trapezoidal: Area
- $\approx \frac{160}{2(8)} [0 + 2(50) + 2(54) + 2(82) + 2(82) + 2(73) + 2(75) + 2(80) + 0] = 9920$
- sq ft

(b) Simpson's: Area $\approx \frac{160}{3(8)} [0 + 4(50) + 2(54) + 4(82) + 2(82) + 4(73) + 2(75) + 4(80) + 0] = 10,413\frac{1}{3}$ sq ft

16. Point of equilibrium:
- $1000 - 0.4x^2 = 42x$

$x = 20, p = 840$

$(P_0, x_0) = (840, 20)$

$$\text{Consumer surplus} = \int_0^{20} [(1000 - 0.4x^2) - 840] dx = 2133.33$$

$$\text{Producer surplus} = \int_0^{20} [840 - 42x] dx = 8400$$