Chart / Misc Answers

AP® CALCULUS BC
2004 SCORING GUIDELINES (Form B)

Question 3

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

(a) Midpoint Riemann sum is $10\cdot[v(5) + v(15) + v(25) + v(35)]$ $= 10\cdot[9.2 + 7.0 + 2.4 + 4.3] = 229$ The integral gives the total distance in miles that the plane flies during the 40 minutes.	3 : { 1 : $v(5) + v(15) + v(25) + v(35)$; 1 : answer; 1 : meaning with units }
(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t.	2 : { 1 : two instances; 1 : justification }
(c) $f'(23) = -0.407$ or -0.408 miles per minute^2	1 : answer with units
(d) Average velocity $= \frac{1}{40}\int_0^{40} f(t)\,dt$ $= 5.916$ miles per minute	3 : { 1 : limits; 1 : integrand; 1 : answer }

AP® CALCULUS BC
2003 SCORING GUIDELINES (Form B)

Question 3

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

(a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

(c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

(a) $\frac{1}{360}\int_0^{360}\frac{B(x)}{2}\,dx$	2 : { 1 : limits and constant; 1 : integrand }
(b) $\frac{1}{360}\left[120\left(\frac{B(60)}{2} + \frac{B(180)}{2} + \frac{B(300)}{2}\right)\right] =$ $\frac{1}{360}[60(30 + 30 + 24)] = 14$	2 : { 1 : $B(60) + B(180) + B(300)$; 1 : answer }
(c) $\frac{B(x)}{2}$ is the radius, so $\pi\left(\frac{B(x)}{2}\right)^2$ is the area of the cross section at x. The expression is the volume in mm^3 of the blood vessel between 125 mm and 275 mm from the end of the vessel.	2 : { 1 : volume in mm^3; 1 : between $x = 125$ and $x = 275$ }
(d) By the MVT, $B'(c_1) = 0$ for some c_1 in $(60, 180)$ and $B'(c_2) = 0$ for some c_2 in $(240, 360)$. The MVT applied to $B'(x)$ shows that $B''(x) = 0$ for some x in the interval (c_1, c_2).	3 : { 2 : explains why there are two values of x where $B'(x)$ has the same value; 1 : explains why that means $B''(x) = 0$ for $0 < x < 360$ } Note: max 1/3 if only explains why $B'(x) = 0$ at some x in $(0, 360)$.

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2007 SCORING GUIDELINES

Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.	$2: \begin{cases} 1: \text{estimate} \\ 1: \text{conclusion with reason} \end{cases}$
(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$ $\left.\frac{dV}{dt}\right\vert_{t=5} = 4\pi(30)^2 2 = 7200\pi$ ft^3/min	$3: \begin{cases} 2: \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$ $= 19.3$ ft $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.	$2: \begin{cases} 1: \text{approximation} \\ 1: \text{explanation} \end{cases}$
(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.	1 : conclusion with reason
Units of ft^3/min in part (b) and ft in part (c)	1 : units in (b) and (c)

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2008 SCORING GUIDELINES

Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8$ people per hour	$2: \begin{cases} 1: \text{estimate} \\ 1: \text{units} \end{cases}$
(b) The average number of people waiting in line during the first 4 hours is approximately $\frac{1}{4}\left(\frac{L(0)+L(1)}{2}(1-0) + \frac{L(1)+L(3)}{2}(3-1) + \frac{L(3)+L(4)}{2}(4-3)\right)$ $= 155.25$ people	$2: \begin{cases} 1: \text{trapezoidal sum} \\ 1: \text{answer} \end{cases}$
(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.	$3: \begin{cases} 1: \text{considers change in sign of } L' \\ 1: \text{analysis} \\ 1: \text{conclusion} \end{cases}$
OR	OR
The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$. [*Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.*]	$3: \begin{cases} 1: \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1: \text{analysis} \\ 1: \text{conclusion} \end{cases}$
(d) $\int_0^3 r(t)\,dt = 972.784$ There were approximately 973 tickets sold by 3 P.M.	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{limits and answer} \end{cases}$

Question 2

For time $t \ge 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{dg}{dt} = \frac{dg}{dx}\cdot\frac{dx}{dt};\ \ \frac{dx}{dt} = r(t)$

$\left.\frac{dg}{dt}\right|_{t=2} = \left.\frac{dg}{dx}\right|_{x=206.370}\cdot r(2)$

$= (0.050)(120) = 6$ liters/hour

$3 : \begin{cases} 2 : \text{uses chain rule} \\ 1 : \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$x(T) = \int_0^T r(t)\,dt = 10.794097$ kilometers

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4 : \begin{cases} 1 : \text{equation } r(t) = 80 \\ 2 : \text{distance integral} \\ 1 : \text{answer} \end{cases}$

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

(a) $\frac{(0+7)}{2}\cdot 8 + \frac{(7+8)}{2}\cdot 6 + \frac{(8+2)}{2}\cdot 8 + \frac{(2+0)}{2}\cdot 2$

$= 115 \text{ ft}^2$

1 : trapezoidal approximation

(b) $\frac{1}{120}\int_0^{120} 115v(t)\,dt$

$= 1807.169$ or $1807.170 \text{ ft}^3/\text{min}$

$3 : \begin{cases} 1 : \text{limits and average value constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230$ or 122.231 ft^2

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is

$\frac{1}{20}\int_{40}^{60} C\cdot v(t)\,dt = 2181.912$ or $2181.913 \text{ ft}^3/\text{min}$.

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100 \text{ ft}^3/\text{min}$.

$3 : \begin{cases} 1 : \text{volumetric flow integral} \\ 1 : \text{average volumetric flow} \\ 1 : \text{answer with reason} \end{cases}$

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2005 SCORING GUIDELINES (Form B)

Question 5

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

(a) $2yy' = y + xy'$ $(2y - x)y' = y$ $y' = \frac{y}{2y - x}$	2 : { 1 : implicit differentiation 1 : solves for y'
(b) $\frac{y}{2y - x} = \frac{1}{2}$ $2y = 2y - x$ $x = 0$ $y = \pm\sqrt{2}$ $(0, \sqrt{2}), (0, -\sqrt{2})$	2 : { 1 : $\frac{y}{2y - x} = \frac{1}{2}$ 1 : answer
(c) $\frac{y}{2y - x} = 0$ $y = 0$ The curve has no horizontal tangent since $0^2 \neq 2 + x \cdot 0$ for any x.	2 : { 1 : $y = 0$ 1 : explanation
(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \frac{7}{3}$. $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{y}{2y - x} \cdot \frac{dx}{dt}$ At $t = 5$, $6 = \frac{3}{6 - \frac{7}{3}} \cdot \frac{dx}{dt} = \frac{9}{11} \cdot \frac{dx}{dt}$ $\left.\frac{dx}{dt}\right\|_{t=5} = \frac{22}{3}$	3 : { 1 : solves for x 1 : chain rule 1 : answer

AP® CALCULUS BC
2006 SCORING GUIDELINES (Form B)

Question 5

Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Let g be a function with $g(4) = 1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y).$$

(a) Find $y = f(x)$.

(b) Given that $g(4) = 1$, find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to\infty} g'(x)$. (It is not necessary to solve for $g(x)$ or to show how you arrived at your answers.)

(c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for $g(x)$.)

(a) $\frac{dy}{dx} = 2y(3 - x)$ $\frac{1}{y}\,dy = 2(3 - x)\,dx$ $\ln\|y\| = 6x - x^2 + C$ $0 = 24 - 16 + C$ $C = -8$ $\ln\|y\| = 6x - x^2 - 8$ $y = e^{6x - x^2 - 8}$ for $-\infty < x < \infty$	5 : { 1 : separates variables 1 : antiderivatives 1 : constant of integration 1 : uses initial condition 1 : solution Note: max 2/5 [1-1-0-0-0] if no constant of integration Note: 0/5 if no separation of variables
(b) $\lim_{x\to\infty} g(x) = 3$ $\lim_{x\to\infty} g'(x) = 0$	2 : { 1 : $\lim_{x\to\infty} g(x) = 3$ 1 : $\lim_{x\to\infty} g'(x) = 0$
(c) $\frac{d^2y}{dx^2} = (6 - 4y)\frac{dy}{dx}$ Because $\frac{dy}{dx} \neq 0$ at any point on the graph of g, the concavity only changes sign at $y = \frac{3}{2}$, half the carrying capacity. $\left.\frac{dy}{dx}\right\|_{y=3/2} = 2\left(\frac{3}{2}\right)\left(3 - \frac{3}{2}\right) = \frac{9}{2}$	2 : { 1 : $y = \frac{3}{2}$ 1 : $\left.\frac{dy}{dx}\right\|_{y=3/2}$

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2008 SCORING GUIDELINES

Question 6

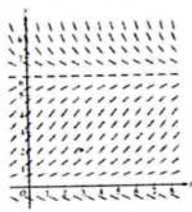
Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.
(Note: Use the axes provided in the exam booklet.)

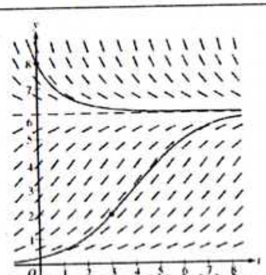
(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \ge 0$?



(a)



$2: \begin{cases} 1: \text{solution curve through } (0,8) \\ 1: \text{solution curve through } (3,2) \end{cases}$

(b) $f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$

$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$

$2: \begin{cases} 1: \text{Euler's method with two steps} \\ 1: \text{approximation of } f(1) \end{cases}$

(c) $\frac{d^2y}{dt^2} = \frac{1}{8}\frac{dy}{dt}(6 - y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$

$f(0) = 8;\ f'(0) = \left.\frac{dy}{dt}\right|_{t=0} = \frac{8}{8}(6 - 8) = -2;$ and

$f''(0) = \left.\frac{d^2y}{dt^2}\right|_{t=0} = \frac{1}{8}(-2)(-2) + \frac{8}{8}(2) = \frac{5}{2}$

The second-degree Taylor polynomial for f about $t = 0$ is $P_2(t) = 8 - 2t + \frac{5}{4}t^2$.

$f(1) \approx P_2(1) = \frac{29}{4}$

$4: \begin{cases} 2: \frac{d^2y}{dt^2} \\ 1: \text{second-degree Taylor polynomial} \\ 1: \text{approximation of } f(1) \end{cases}$

(d) The range of f for $t \ge 0$ is $6 < y \le 8$.

1 : answer

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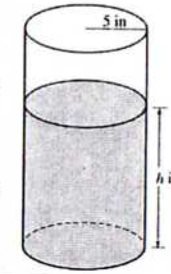
2003 SCORING GUIDELINES

Question 5

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

(a) $V = 25\pi h$

$\frac{dV}{dt} = 25\pi\frac{dh}{dt} = -5\pi\sqrt{h}$

$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$

$3: \begin{cases} 1: \frac{dV}{dt} = -5\pi\sqrt{h} \\ 1: \text{computes } \frac{dV}{dt} \\ 1: \text{shows result} \end{cases}$

(b) $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$\frac{1}{\sqrt{h}}dh = -\frac{1}{5}dt$

$2\sqrt{h} = -\frac{1}{5}t + C$

$2\sqrt{17} = 0 + C$

$h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$

$5: \begin{cases} 1: \text{separates variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition } h = 17 \text{ when } t = 0 \\ 1: \text{solves for } h \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(c) $\left(-\frac{1}{10}t + \sqrt{17}\right)^2 = 0$

$t = 10\sqrt{17}$

1 : answer