A function is continuous at $x=a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

Implies $\Rightarrow$ ① $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

② $\lim_{x \to a} f(x)$ exists

③ $f(a)$ exists

Jump discontinuity

If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

jump discont. @ $x=a$.

Removable discont

If $\lim_{x \to a^-} f(x) = \lim_{x \to a} f(x) \neq f(a)$

then removable discont @ $x=a$.

Infinite discont (vert asymptote)

$\lim_{x \to a^-} f(x) = \pm\infty$

$f(a)$ DNE

$\lim_{x \to a^+} f(x) = \pm\infty$

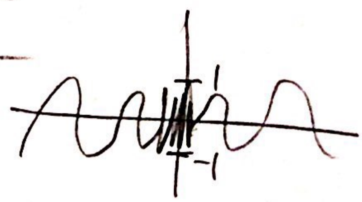
then infinite discont @ $x=a$.

Oscillating discont

$\lim_{x \to a} f(x)$ DNE

Ex. $y = \cos\left(\frac{1}{x}\right)$

$y = \sin\left(\frac{1}{x}\right)$



$$f(x) = \begin{cases} 3x-1 & x<2 \\ 4 & x=2 \\ 2x+1 & x>2 \end{cases}$$

Prove whether or not $f(x)$ is cont @ $x=2$. If not, classify discont.

$\lim_{x \to 2^-} 3x-1 = 5$

$\lim_{x \to 2^+} 2x+1 = 5$

$\lim_{x \to 2} f(x) = 5 \neq f(2) = 4$

So not continuous

Removable discont

$$f(x) = \begin{cases} \dfrac{x^2-4}{x+2} & x \neq -2 \\ a & x = -2 \end{cases}$$

Find the value of a to make f(x) cont.

$$\lim_{x \to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}} = -4 \qquad a = -4$$

$$f(x)=\begin{cases} cx^2-x+4 & x<-1 \\ d & x=-1 \\ 3x+4 & x>-1 \end{cases}$$

Find values of c and d to make f(x) cont at $x=-1$.

$c=-4 \quad d=1$

$$\lim_{x\to -1^-} cx^2-x+4 = \boxed{\lim_{x\to -1^+} 3x+4 = d}$$

$c+5=1$

$c=-4$

$\boxed{1=d}$

$$f(x)=\begin{cases} x^2-2x+4 & x\le 1 \\ 6 & x=1 \\ 2x+1 & x>1 \end{cases}$$

Prove if f(x) is cont at $x=1$. If not, classify discont.

$$\lim_{x\to 1^-} x^2-2x+4 \overset{?}{=} \lim_{x\to 1^+} 2x+1 \overset{?}{=} f(1)$$

$3 \overset{?}{=} 3 = f(1)$ ✓ So cont. @ $x=1$

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