Swok #5-43 odd, 53-61 odd, 65, 67, 69, 70

EXERCISES 3.3

Exer. 1–40: Find the derivative.

1 $g(t) = 6t^{5/3}$

2 $h(z) = 8z^{3/2}$

3 $f(s) = 15 - s + 4s^2 - 5s^4$

4 $f(t) = 12 - 3t^4 + 4t^6$

5 $f(x) = 3x^2 + \sqrt[3]{x^4}$

6 $g(x) = x^4 - \sqrt[4]{x^3}$

7 $g(x) = (x^3 - 7)(2x^2 + 3)$

8 $k(x) = (2x^2 - 4x + 1)(6x - 5)$

9 $f(x) = x^{1/2}(x^2 + x - 4)$

10 $h(x) = x^{2/3}(3x^2 - 2x + 5)$

11 $h(r) = r^2(3r^4 - 7r + 2)$

12 $k(v) = v^3(-2v^3 + v - 3)$

13 $g(x) = (8x^2 - 5x)(13x^2 + 4)$

14 $H(z) = (z^5 - 2z^3)(7z^2 + z - 8)$

15 $f(x) = \dfrac{4x - 5}{3x + 2}$

16 $h(x) = \dfrac{8x^2 - 6x + 11}{x - 1}$

17 $h(z) = \dfrac{8 - z + 3z^2}{2 - 9z}$

18 $f(w) = \dfrac{2w}{w^3 - 7}$

19 $G(v) = \dfrac{v^3 - 1}{v^3 + 1}$

20 $f(t) = \dfrac{8t + 15}{t^2 - 2t + 3}$

21 $g(t) = \dfrac{\sqrt[3]{t^2}}{3t - 5}$

22 $f(x) = \dfrac{\sqrt{x}}{2x^2 - 4x + 8}$

23 $f(x) = \dfrac{1}{1 + x + x^2 + x^3}$

24 $p(x) = 1 + \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}$

25 $h(x) = \dfrac{7}{x^2 + 5}$

26 $k(z) = \dfrac{6}{z^2 + z - 1}$

27 $F(t) = t^2 + \dfrac{1}{t^2}$

28 $s(x) = 2x + \dfrac{1}{2x}$

29 $K(s) = (3s)^{-4}$

30 $W(s) = (3s)^4$

31 $h(x) = (5x - 4)^2$

32 $S(w) = (2w + 1)^3$

33 $g(r) = (5r - 4)^{-2}$

34 $S(x) = (3x + 1)^{-2}$

35 $f(t) = \dfrac{3/(5t) - 1}{(2/t^2) + 7}$

36 $N(z) = \dfrac{4/z^2}{(3/z) + 2}$

37 $M(x) = \dfrac{2x^3 - 7x^2 + 4x + 3}{x^2}$

38 $T(z) = \dfrac{5z^4 + z^3 - 2z}{z^3}$

39 $f(x) = \dfrac{3x^2 - 5x + 8}{7}$

40 $h(t) = \dfrac{3t^5 + 2t}{5}$

Exer. 67–70: If f and g are functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = -5$, and $g'(2) = 2$, evaluate the expression.

67 (a) $(f + g)'(2)$ (b) $(f - g)'(2)$ (c) $(4f)'(2)$
(d) $(fg)'(2)$ (e) $(f/g)'(2)$ (f) $(1/f)'(2)$

Exer. 41–44: Solve the equation $D_x\, y = 0$.

41 $y = 2x^3 - 3x^2 - 36x + 4$

42 $y = 4x^3 + 21x^2 - 24x + 11$

43 $y = \dfrac{2x^2 + 3x - 6}{x - 2}$

44 $y = \dfrac{x^2 + 2x + 5}{x + 1}$

Exer. 45–46: Solve the equation $D_x^2\, y = 0$.

45 $y = 6x^4 + 24x^3 - 540x^2 + 7$

46 $y = 6x^5 - 5x^4 - 30x^3 + 11x$

Exer. 47–50: Find dy/dx by (a) using the quotient rule, (b) using the product rule, and (c) simplifying algebraically and using (3.18).

47 $y = \dfrac{3x - 1}{x^2}$

48 $y = \dfrac{x^2 + 1}{x^4}$

49 $y = \dfrac{x^2 - 3x}{\sqrt[3]{x^2}}$

50 $y = \dfrac{2x + 3}{\sqrt{x^3}}$

Exer. 51–52: Find d^2y/dx^2.

51 $y = \dfrac{3x + 4}{x + 1}$

52 $y = \dfrac{x + 3}{2x + 3}$

Exer. 53–54: Find an equation of the tangent line to the graph of f at P.

53 $f(x) = \dfrac{5}{1 + x^2}$; $P(-2, 1)$

54 $f(x) = 3x^2 - 2\sqrt{x}$; $P(4, 44)$

55 Find the x-coordinates of all points on the graph of $y = x^3 + 2x^2 - 4x + 5$ at which the tangent line is
(a) horizontal (b) parallel to the line $2y + 8x = 5$

56 Find the point P on the graph of $y = x^3$ such that the tangent line at P has x-intercept 4.

57 Find the points on the graph of $y = x^{3/2} - x^{1/2}$ at which the tangent line is parallel to the line $y - x = 3$.

58 Find the points on the graph of $y = x^{5/3} + x^{1/3}$ at which the tangent line is perpendicular to the line $2y + x = 7$.

Exer. 59–60: Sketch the graph of the equation and find the vertical tangent lines.

59 $y = \sqrt{x} - 4$

60 $y = x^{1/3} + 2$

61 A weather balloon is released and rises vertically such that its distance $s(t)$ above the ground during the first 10 seconds of flight is given by $s(t) = 6 + 2t + t^2$, where $s(t)$ is in feet and t is in seconds. Find the velocity of the balloon at
(a) $t = 1$, $t = 4$, and $t = 8$
(b) the instant the balloon is 50 feet above the ground

Exer. 65–66: Find equations of the lines through P that are tangent to the graph of the equation.

65 $P(5, 9)$; $y = x^2$

66 $P(3, 1)$; $xy = 4$

69 (a) $(2f - g)'(2)$ (b) $(5f + 3g)'(2)$
(c) $(gg)'(2)$ (d) $\left(\dfrac{1}{f + g}\right)'(2)$

70 (a) $(3f - 2g)'(2$ ... $(5/g)'(2)$
(c) $(6f)'(2)$ (d) $\left(\dfrac{f}{f + g}\right)'(2)$

Exercises 3.3

1. $g'(t) = 6 \cdot \frac{5}{3} \cdot t^{2/3} = 10t^{2/3}$
2. $h'(z) = 8 \cdot \frac{3}{2} \cdot z^{1/2} = 12z^{1/2}$
3. $f'(s) = -1 + 8s - 20s^3$
4. $f'(t) = -12t^3 + 24t^5$
5. $f'(x) = 6x + \frac{4}{3}x^{1/3}$
6. $g'(x) = 4x^3 - \frac{3}{4}x^{-1/4}$
7. $g'(x) = (x^3 - 7)(4x) + (2x^2 + 3)(3x^2) = 10x^4 + 9x^2 - 28x$
8. $k'(x) = (2x^2 - 4x + 1)(6) + (6x - 5)(4x - 4) = 36x^2 - 68x + 26$
9. $f(x) = x^{5/2} + x^{3/2} - 4x^{1/2} \Rightarrow f'(x) = \frac{5}{2}x^{3/2} + \frac{3}{2}x^{1/2} - 2x^{-1/2}$
10. $h(x) = 3x^{8/3} - 2x^{5/3} + 5x^{2/3} \Rightarrow h'(x) = 8x^{5/3} - \frac{10}{3}x^{2/3} + \frac{10}{3}x^{-1/3}$
11. $h(r) = 3r^6 - 7r^3 + 2r^2 \Rightarrow h'(r) = 18r^5 - 21r^2 + 4r$
12. $k(v) = -2v^6 + v^4 - 3v^3 \Rightarrow k'(v) = -12v^5 + 4v^3 - 9v^2$
13. $g'(x) = (8x^2 - 5x)(26x) + (13x^2 + 4)(16x - 5) = 416x^3 - 195x^2 + 64x - 20$
14. $H'(z) = (z^5 - 2z^3)(14z + 1) + (7z^2 + z - 8)(5z^4 - 6z^2)$
$= 49z^6 + 6z^5 - 110z^4 - 8z^3 - 48z^2$

15. $f'(x) = \dfrac{(3x+2)(4) - (4x-5)(3)}{(3x+2)^2} = \dfrac{23}{(3x+2)^2}$
16. $h'(x) = \dfrac{(x-1)(16x-6) - (8x^2 - 6x + 11)(1)}{(x-1)^2} = \dfrac{8x^2 - 16x - 5}{(x-1)^2}$
17. $h'(z) = \dfrac{(2-9z)(-1+6z) - (8 - z + 3z^2)(-9)}{(2-9z)^2} = \dfrac{70 + 12z - 27z^2}{(2-9z)^2}$
18. $f'(w) = \dfrac{(w^3 - 7)(2) - (2w)(3w^2)}{(w^3-7)^2} = \dfrac{-4w^3 - 14}{(w^3-7)^2} = -\dfrac{4w^3 + 14}{(w^3-7)^2}$
19. $G'(v) = \dfrac{(v^3+1)(3v^2) - (v^3-1)(3v^2)}{(v^3+1)^2} = \dfrac{6v^2}{(v^3+1)^2}$
20. $f'(t) = \dfrac{(t^2 - 2t + 3)(8) - (8t + 15)(2t - 2)}{(t^2 - 2t + 3)^2} = \dfrac{-8t^2 - 30t + 54}{(t^2 - 2t + 3)^2}$
21. $g'(t) = \dfrac{(3t-5)(\frac{2}{3}t^{-1/3}) - (t^{2/3})(3)}{(3t-5)^2} = \dfrac{t^{-1/3}[2(3t-5) - 9t]}{3(3t-5)^2} = -\dfrac{3t+10}{3\sqrt[3]{t}(3t-5)^2}$

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22. $f'(x) = \dfrac{(2x^2 - 4x + 8)(\frac{1}{2}x^{-1/2}) - (x^{1/2})(4x - 4)}{(2x^2 - 4x + 8)^2}$
$= \dfrac{x^{-1/2}[(x^2 - 2x + 4) - x(4x - 4)]}{(2x^2 - 4x + 8)^2} = \dfrac{-3x^2 + 2x + 4}{\sqrt{x}(2x^2 - 4x + 8)^2}$
23. By the reciprocal rule, $f'(x) = -\dfrac{1 + 2x + 3x^2}{(1 + x + x^2 + x^3)^2}$.
24. $p(x) = 1 + x^{-1} + x^{-2} + x^{-3} \Rightarrow p'(x) = -x^{-2} + (-2)x^{-3} + (-3)x^{-4} =$
$-\dfrac{1}{x^2} - \dfrac{2}{x^3} - \dfrac{3}{x^4}$
25. $h'(x) = 7\left[-\dfrac{2x}{(x^2+5)^2}\right] = \dfrac{-14x}{(x^2+5)^2}$
26. $k'(z) = 6\left[-\dfrac{2z+1}{(z^2+z-1)^2}\right]$
27. $F'(t) = 2t - \dfrac{2t}{(t^2)^2} = 2t - \dfrac{2}{t^3}$
28. $s'(z) = 2 - \dfrac{2}{(2z)^2} = 2 - \dfrac{1}{2z^2}$
29. $K'(s) = D_s(3^{-4}s^{-4}) = 3^{-4}(-4s^{-5}) = -\frac{4}{81}s^{-5}$
30. $W'(s) = D_s(3^4s^4) = 3^4(4s^3) = 324s^3$
31. $h'(x) = D_x(25x^2 - 40x + 16) = 50x - 40 = 10(5x - 4)$
32. $S'(w) = D_w(8w^3 + 12w^2 + 6w + 1) = 24w^2 + 24w + 6 = 6(4w^2 + 4w + 1)$
33. $g'(r) = D_r\left[\dfrac{1}{(5r-4)^2}\right] = -\dfrac{10(5r-4)}{(5r-4)^4} = \dfrac{-10}{(5r-4)^3}$,
where we used Exercise 31 for $D_r\left[(5r-4)^2\right]$.
34. $S'(x) = S_x\left[\dfrac{1}{(3x+1)^2}\right] = S_x\left[\dfrac{1}{9x^2 + 6x + 1}\right] =$
$-\dfrac{18x + 6}{(9x^2 + 6x + 1)^2} = \dfrac{-6(3x+1)}{(3x+1)^4} = \dfrac{-6}{(3x+1)^3}$

35. $f'(t) = D_t\left(\dfrac{\frac{3}{5}t - t^2}{2 + 7t^2}\right) = \dfrac{(2 + 7t^2)(\frac{3}{5} - 2t) - (\frac{3}{5}t - t^2)(14t)}{(2 + 7t^2)^2} =$
$\dfrac{-\frac{21}{5}t^2 - 4t + \frac{6}{5}}{(2 + 7t^2)^2} = \dfrac{6 - 20t - }{5(2 + 7t}$
36. $N'(z) = D_z\left(\dfrac{4}{3z + 2z^2}\right) = 4\left[-\dfrac{3 + 4z}{(3z + 2z^2)^2}\right] = \dfrac{-4(3 + 4z)}{(3z + 2z^2)^2}$
37. $M'(x) = D_x(2x - 7 + 4x^{-1} + 3x^{-2}) = 2 - 4x^{-2} - 6x^{-3} = 2 - \dfrac{4}{x^2} - \dfrac{6}{x^3}$
38. $T'(z) = D_z(5z + 1 - 2z^{-2}) = 5 + 4z^{-3}$
39. $f'(x) = \frac{1}{7}D_x(3x^2 - 5x + 8) = \frac{1}{7}(6x - 5)$
40. $h'(t) = \frac{1}{5}D_t(3t^5 + 2t) = \frac{1}{5}(15t^4 + 2)$
41. $D_x y = 6x^2 - 6x - 36 = 6(x - 3)(x + 2)$; $D_x y = 0 \Rightarrow x = -2, 3$.
42. $D_x y = 12x^2 + 42x - 24 = 6(2x - 1)(x + 4)$; $D_x y = 0 \Rightarrow x = -4, \frac{1}{2}$.
43. $D_x y = \dfrac{(x - 2)(4x + 3) - (2x^2 + 3x - 6)(1)}{(x - 2)^2} = \dfrac{2x^2 - 8x}{(x - 2)^2} = \dfrac{2x(x - 4)}{(x - 2)^2}$;

X=0
X=4

53. $f'(x) = 5\left[-\dfrac{2x}{(1 + x^2)^2}\right] = \dfrac{-10x}{(1 + x^2)^2}$, $f'(-2) = \frac{20}{25}$;
$y - 1 = \frac{4}{5}(x + 2)$ or $y = \frac{4}{5}x + \frac{13}{5}$
54. $f'(x) = 6x - x^{-1/2}$, $f'(4) = \frac{47}{2}$; $y - 44 = \frac{47}{2}(x - 4)$ or $y = \frac{47}{2}x - 50$
55. (a) $f'(x) = 3x^2 + 4x - 4 = (3x - 2)(x + 2)$; $f'(x) = 0 \Rightarrow x = \frac{2}{3}, -2$
(b) The given line has slope -4. $f'(x) = -4 \Rightarrow$
$3x^2 + 4x - 4 = -4 \Rightarrow 3x^2 + 4x = 0 \Rightarrow x(3x + 4) = 0 \Rightarrow x = 0, -\frac{4}{3}$
56. Let $f(x) = x^3$. $f'(a) = 3a^2$ and the tangent line equation is $(y - a^3) = 3a^2(x - a)$.
If this line has x-intercept 4, then $(4, 0)$ must satisfy its equation.
Thus, $(0 - a^3) = 3a^2(4 - a) \Rightarrow 2a^2(a - 6) = 0 \Rightarrow a = 0, 6$.
If a were 0, then P would have x-intercept 0; so a must be 6 and P is $(6, 216)$.

57. The slope of the given line is 1. $y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \dfrac{3x - 1}{2\sqrt{x}}$.
Now, $\dfrac{3x - 1}{2\sqrt{x}} = 1 \Rightarrow 9x^2 - 6x + 1 = 4x \Rightarrow (9x - 1)(x - 1) = 0 \Rightarrow x = \frac{1}{9}, 1$.
The points are $(\frac{1}{9}, -\frac{8}{27})$ and $(1, 0)$.
58. The given line has slope $-\frac{1}{2}$, so the desired slope is 2.
$y' = \frac{5}{3}x^{2/3} + \frac{1}{3}x^{-2/3} = 2 \Rightarrow \frac{5}{3}x^{4/3} + \frac{1}{3} - 2x^{2/3} = 0 \Rightarrow$
$5(x^{2/3})^2 - 6(x^{2/3}) + 1 = 0 \Rightarrow (5x^{2/3} - 1)(x^{2/3} - 1) = 0 \Rightarrow x = \pm(\frac{1}{5})^{3/2}, \pm 1$.
The four points are $(\pm\frac{1}{5\sqrt{5}}, \pm\frac{26}{25\sqrt{5}})$ and $(\pm 1, \pm 2)$.
59. $y' = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$; $\lim_{x \to 0^+}\left|\dfrac{1}{2\sqrt{x}}\right| = \infty \Rightarrow$ vertical tangent line at $x = 0$.

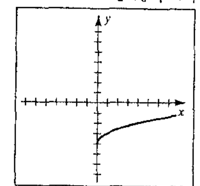

Figure 59

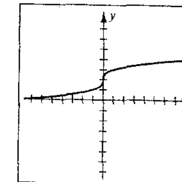

Figure 60

60. $y' = \frac{1}{3}x^{-2/3} = \dfrac{1}{3\sqrt[3]{x^2}}$; $\lim_{x \to 0}\left|\dfrac{1}{3\sqrt[3]{x^2}}\right| = \infty \Rightarrow$ vertical tangent line at $x = 0$.

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61. (a) $v(t) = s'(t) = 2 + 2t$; $v(1) = 4$, $v(4) = 10$, and $v(8) = 18$ (ft/sec)
(b) $s(t) = 50 \Rightarrow t^2 + 2t - 44 = 0 \Rightarrow t = -1 \pm 3\sqrt{5}$;
$v(-1 + 3\sqrt{5}) = 6\sqrt{5}$ ft/sec ≈ 13.4 ft/sec

65. Let (a, a^2) be the point of tangency. The slope of the tangent line is $2a$ and
$(a^2 - 9) = 2a(a - 5) \Rightarrow a^2 - 10a + 9 = 0 \Rightarrow a = 1, 9$. Thus, there are two such
lines: $(y - 9) = 2(x - 5)$ and $(y - 9) = 18(x - 5)$ or equivalently,
$y = 2x - 1$ and $y = 18x - 81$.

67. (a) $(f + g)'(2) = f'(2) + g'(2) = -1 + 2 = 1$
(b) $(f - g)'(2) = f'(2) - g'(2) = -1 - 2 = -3$
(c) $(4f)'(2) = 4f'(2) = 4(-1) = -4$
(d) $(fg)'(2) = f(2)g'(2) + g(2)f'(2) = (3)(2) + (-5)(-1) = 11$
(e) $\left(\frac{f}{g}\right)'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \dfrac{(-5)(-1) - (3)(2)}{(-5)^2} = -\frac{1}{25}$
(f) $(1/f)'(2) = -\dfrac{f'(2)}{[f(2)]^2} = -\dfrac{-1}{3^2} = \frac{1}{9}$
68. (a) $(g - f)'(2) = g'(2) - f'(2) = 2 - (-1) = 3$
(b) $\left(\frac{g}{f}\right)'(2) = \dfrac{f(2)g'(2) - g(2)f'(2)}{[f(2)]^2} = \dfrac{(3)(2) - (-5)(-1)}{(3)^2} = \frac{1}{9}$
(c) $(4g)'(2) = 4g'(2) = 4(2) = 8$
(d) $(ff)'(2) = f(2)f'(2) + f(2)f'(2) = 2f(2)f'(2) = 2(3)(-1) = -6$
69. (a) $(2f - g)'(2) = 2f'(2) - g'(2) = 2(-1) - 2 = -4$
(b) $(5f + 3g)'(2) = 5f'(2) + 3g'(2) = (5)(-1) + (3)(2) = 1$
(c) $(gg)'(2) = g(2)g'(2) + g(2)g'(2) = 2g(2)g'(2) = 2(-5)(2) = -20$
(d) $\left(\dfrac{1}{f + g}\right)'(2) = -\dfrac{(f + g)'(2)}{[(f + g)(2)]^2} = -\dfrac{f'(2) + g'(2)}{[f(2) + g(2)]^2} = -\dfrac{-1 + 2}{(-2)^2} = -\frac{1}{4}$

70. a) -7 b) $-\frac{2}{5}$ c) -6 d) $-\frac{1}{4}$